MADURAI KAMARAJ UNIVERSITY
(University with Potential for Excellence)
DIRECTORATE OF DISTANCE EDUCATION

B.Sc., Physics

PAPER - III

ELECTRICITY AND ELECTROMAGNETISM

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B.Sc. Physics

Paper - III

ELECTRICITY AND ELECTROMAGNETISM

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Printed at Vimala Note book
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Paper –III – Electricity and Electromagnetism

Unit I

Coulomb's law – Gauss law – Its proof and applications – Electric field due to a charged sphere – Electric field due to a plain sheet of charged conductor – Coulomb's theorem – Mechanical force on the surface of the charged conductor – Electric field, Electric potential. Relation between them – Electric field due to electric dipole, on the axial line and equatorial line and equatorial line – Potential due to charged conductor – Capacitance – Principal Expressions for the capacitance – Spherical capacitor – Cylindrical capacitor –Parallel plate capacitor with and without the dielectric – Energy of capacitor – Loss of energy due to sharing of charges – Types of capacitors, fixed capacitor, variable capacitor. Electrolytic capacitor and sliding capacitor.

Unit II

Kirchoff's laws – Application to wheatstone's bridge – Sensitiveness of bridge – Carey Foster's bridge – Determination of the resistance – Theory – Principle of potentiometer – Determination of internal resisitance of the cell using potentiometer -Calibration of ammeter and voltmeter – Low & high range – Seeback effect – Thermo e.m.f – Neutral temperature – Temperature of inversion – Law of intermediate metals – Law of intermediate temperature – Measurement of e.m.f of a thermocouple with a potentiometer – Peltier effect – Peltier coefficient – Thomson coefficient – Thermoelectric power.

Unit III

Biot-savat's law – Its application – Long straight wire of infinite length – Ampere's theorem – Magnetic field at the center of current carrying circular coil – Solenoid – Ballistic galvanometer – Theory – Damping correction – comparison between deadbeat and a periodic galvanometer Determination of absolute capacity of a conductor. Comparison of capacitance using B.G. (theory & experiment) – Farady's laws of electromagnetic induction – Lenz's law – Self –inductance – Energy stored in an inductor – Self-inductance by Raylegh's bridge method –Mutual inductance Determination using B.G theory – Coefficient of coupling Eddy current

Unit IV

Growth and decay of current in LR circuit – Growth and decay of charges in CR circuit - Growth and decay of charges in a circuit with inductance, Capacitance and resistance in series – Determination of High resistance by leakage (B.G) – Mean value of alternating e.m.f –RMS value of alternating current/voltage – Alternating current applied to LR, CR and LCR circuits – Series resonance circuit – Parallel resonance circuit – Power in an A.C. circuit – Wattless current – Power factor – Q – factor- Choke – Skin effect – A.C. Bridges – Maxwell's bridge – Anderson's bridge and Owen's bridge.

Unit V

Definition of B.H. M and magnetic susceptibility – Magnetics materials and magnetization – Hysterics – Work done in taking unit volume of magnetic materials through complete cycle of magnetization - Area of Hysteresis loop – Ballistic method Ferro magnets, Ferric magnets and determination of susceptibility – Guoy's method - Derivations of Maxwell's equations – Types of currents – Displacement current – Significance of displacement of current – Maxwell's equations n material media and free space – Electromagnetic waves in free space – Electromagnetic waves in isotropic non conducting media.

Text Books

1. Mechanics and Properties of Mater by R. Murugeasan – Rtd Prof. Vivekanada College, Thiruvedagam West.
2. Electric and Magnetism 20th revised edition – Brijlal and Subramaniyam S. chand and Co. 2007

Paper –III – Electricity and Electromagnetism

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Unit I: Coulomb's Law

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1.2. Objectives.
1.3. Coulomb's law
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  1.4.1. Differential form of Gauss Law
1.5. Its proof and applications
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1.6. Coulomb's theorem
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1.12. Energy of capacitor – Loss of energy due to sharing of charges
1.13. Loss of energy on sharing of charges between two capacitors
1.14. Types of capacitors, fixed capacitor, and variable capacitor.
1.15. Summary.
1.16. Unit – end exercises.
1.17. Answers to check your progress and problems for discussion.
1.18. Suggested readings.

1.1 Introduction :

Electrostatics is the branch of Physics that deals with the phenomena and properties of stationary (without acceleration) electric charges. Electrostatics is the study of the electrical fields surrounding electrical charges and the resulting forces between charged surfaces. The resulting interactions are entirely dependent on the charges and their relative positions and not by their motion. Coulomb's law is a law of physics describing the electrostatic interaction between electrically charged particles. It was essential for the development of the theory of electromagnetism.

The detailed study of Gauss's law deals with electric field due to continuous charge distributions. An electrostatic field is an electric field produced by static electric charges. Electrostatic field plays an important role in modern design of electromagnetic devices whenever a strong electric field appears. For example, an electric field is of paramount importance for the design of X-ray devices, lightning protection equipment and high-voltage components of electric power transmission systems. This is important for high-power applications. In the area of solid-state electronics. This situation seems to be similar in cathode ray tubes, liquid crystal display, touch pads etc. In order to reach a high level of knowledge, understanding the fundamental theory behind electrostatic field that will be presented in this chapter.

1.2 Objectives :

After going through this unit, you will be able to:

- distinguish between two types of charges.
- infer that any electric charge is always an integral multiple of the charge of the electron.
- use coulomb's law to find the electrostatic force between the charges
- write the relation between the electric flux and the charge enclosed within the surface.
- use the Gauss law to compute the electric fields in case of spherical and planar symmetry.
- compute the electric potential at a point due to a single charge.
- compute the electric field due to a dipole at a point as the axial line & equatorial line.
- relate the electric potential & electric filed and therefore compute the electric field at a point knowing the electric potential.
- define the capacitance of a capacitor.
- calculate the energy stored in a capacitor.

1.3 Coulomb's Law :

The study of charges is called electricity. The study of charges at rest is called electrostatics. The study of charges in motion is called current electricity.

Charges are of the two types:

(1) Positive charges (2) Negative charges

Similar charges repel and opposite charges attract.

So far as their electrostatic behaviour is concerned, materials are divided into two categories: Conductors of electricity and insulators (dielectrics). Bodies which allow the charge or electricity to pass through them are called conductors, e.g., metals, human body, earth, graphite, charcoal etc. Bodies which do not allow the charge or electricity to pass through them are called insulators, e.g., glass, mica, ebonite, plastic etc.

Statement: The force between two point charges is directly proportional to the product of the charges, and inversely proportional to the square of the distance between them.

Explanation:

Let q_1 and q_2 be two point charges situated at A and B separated by a distance r (Fig 1.1). $\hat{r}$ is the unit vector in the direction from A to B. Then, the force (F_{12}) exerted by charge q_1 on charge q_2 is

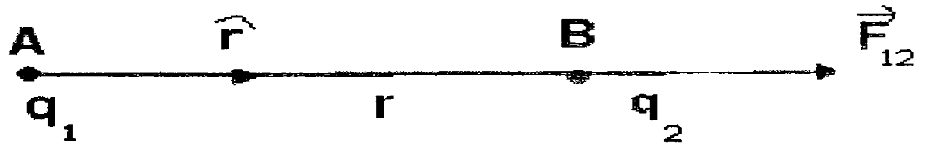


Fig 1.1

$$F_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r} \qquad .....(1)$$

Here, the charges are situated in vacuum.

ε_0 is called the permittivity of free space (i.e., vacuum).

$$\varepsilon_0 = 8.85418\ X\ 10^{-12}(C^2N^{-1}m^{-2}(or\ F\ m^{-1}).$$

And $1/(4\pi\varepsilon_0) = 9\ X 10^9 Nm^2C^{-2}$

Suppose the charges are situated in medium of permittivity ε. Then, the force between the charges is

$$F_{12}(m) = \frac{1}{4\pi}\frac{q_1 q_2}{r^2}\hat{r} \qquad .....(2)$$

Dividing Eq.(1) by Eq.(2), we get

$$\frac{F_{12}}{F_{12(m)}} = \frac{\varepsilon}{\varepsilon_0} = \varepsilon_r \qquad ....(3)$$

ε_r is called the relative permittivity of the medium. It is defined as the ratio of the permittivity of the medium to that of free space.

(An older name for ε_r is the dielectric constant of the material).

The value of ε_r for air is 1.

In Eq. (1), if $q_1 = q_2 = 1$ and r = 1, we have

$$F = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{q_1 q_2}{r^2} = (9 * 10^9)\frac{1 * 1}{1^2} = 9\,X\,10^9 N.$$

The SI unit of charge is the coulomb.

A coulomb is defined as the quantity of charge which, when placed at a distance of 1 metre in air or vacuum from an equal and similar charge experiences a repulsive force of $9\,X\,10^9 N$.

Check your progress:
1. What will happen to the electro static force if the distance between the charges is doubled? Ans:-- -- -- --

1.4 Gauss's Law

Statement: The total electric flux (φ) of the electric field E over any closed surface is equal to $(1/\varepsilon_0)$ l times the total net charge (q) enclosed by the surface.

$$\varphi = \oint E.dS = \frac{q}{\varepsilon_0} \qquad ....(1)$$

Here, ε_0 is the permittivity of the free space.

Explanation: This law relates the flux through any closed surface and the net charge enclosed within the surface. Here q is the net charge inside the closed surface. This closed hypothetical surface is called Gaussian surface. Gauss' law tells us that the flux of E through a closed surface S depends only on the value of the net charge inside the surface and not on the location of the charges. Charges outside the surface will not contribute to φ .Eqn.(1) holds only when the charges are lying in vacuum or air.

Gauss Law is the converse of Coulomb's law. With the help of Coulomb's law, we can calculate E for a given charge. Gauss's law enables us to determine the charge provided if E is known.

Proof

(i) For a charge inside the closed surface.

Consider a single point charge +q located at a point O inside a closed surface S (Fig .1.2). Let dS be a small area element at a distance r from q.

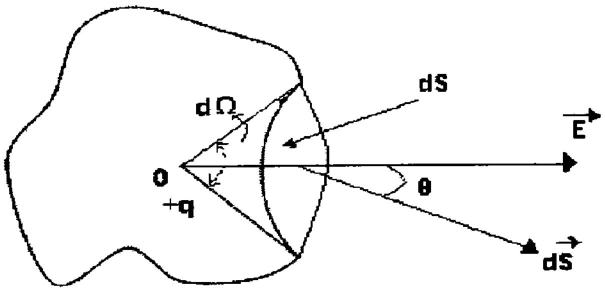


Fig.1.2

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\,\hat{r}$$

The flux through the area dS is given by

$$d\varphi = E.dS$$

$$= E\, dS \cos\theta$$

$$= \left(\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\right) dS\cos\theta$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{dS\cos\theta}{r^2}\right)$$

But $\frac{dS\cos\theta}{r^2}$ = dΩ = Solid angle subtended by the area dS at O.

$$d\varphi = \frac{q}{4\pi\varepsilon_0}d\Omega$$

The total flux through the entire closed surface S is given by

$$\varphi = \oint d\varphi = \frac{q}{4\pi\varepsilon_0}\oint d\Omega = \frac{q}{4\pi\varepsilon_0} X\ 4\pi$$

Here, $\oint d\Omega$ = total solid angle subtended by S at O = 4π

$$\varphi = \frac{q}{\varepsilon_0}$$

Gauss's law holds even if there are a number of charges $q_1, q_2 \ldots\ldots q_n$ enclosed by a surface S (Fig.1.3). We know from the principle of superposition that the electric field due to a number of charges is the vector sum of their individual fields.

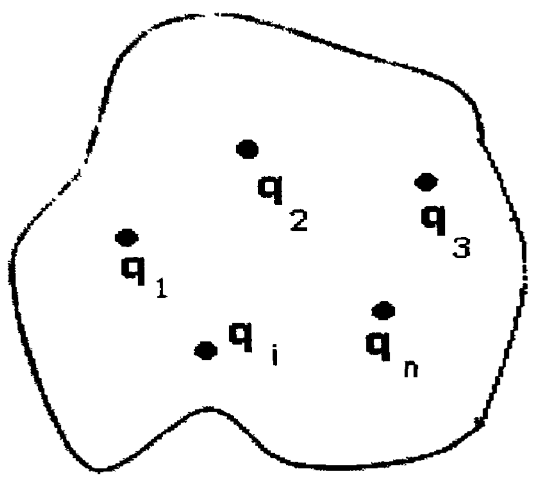


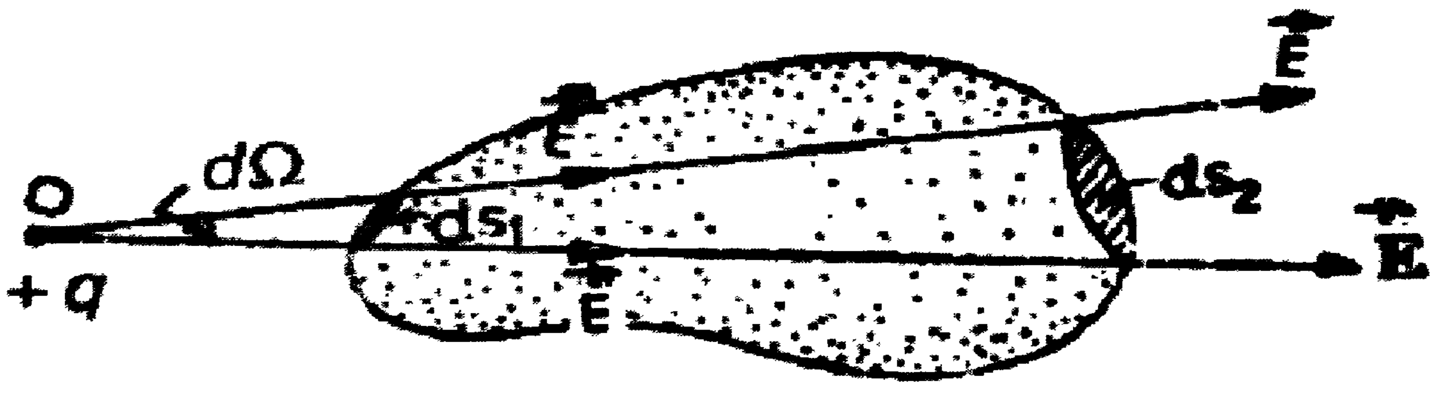
Fig 1.3

$$\varphi = \oint E.dS = \frac{1}{\varepsilon_0}\sum_{i-1}^{n} q_i = \frac{Q}{\varepsilon_0}$$

Here, Q is the total charge inside the surface.

(ii) For a charge outside the closed surface.

Consider a point charge +q situated at O outside the closed surface (Fig 1.4). Let an elementary cone form O with small solid angle $d\Omega$ cut the closed surface at two elements of area dS_1 and dS_2. Magnitude of flux through dS_1 and dS_2 equal. Flux through dS_1 is an inward flux. Flux through dS_2 is an outward flux.



Therefore, total flux through dS_1 and dS_2 $= \frac{-q}{4\pi\varepsilon_0} d\Omega + \frac{q}{4\pi\varepsilon_0} d\Omega = 0$

The entire closed surface can be considered to be made of pairs of elements like dS_1 and dS_2. Thus the total flux, due to a charge outside is zero.

1.4.1 Differential form of Gauss Law

Suppose the charge is distributed over a volume. Let ρ be the charge density. Then the total charge within the closed surface enclosing the volume is given by

$$Q = \int \rho \, dV \qquad .....(1)$$

We can write integral form of Gauss law as

$$\oint E.dS = \frac{1}{\varepsilon_0}\int \rho \, dV \; .....(2)$$

By Gauss divergence theorem,

$$\oint E.dS = \int (\nabla.E) \, dV \; .....(3)$$

Comparing Eq.(2) and Eq.(3)

$$\int (\nabla.E) \, dV = \frac{1}{\varepsilon_0}\int \rho \, dV \quad ......(4)$$

Since this is true for any volume v, integrands must be equal

$$\nabla.E = \frac{\rho}{\varepsilon_0}$$

1.5 Applications of Gauss's Law

1.5.1 Electric field due to a Uniformly Charged Sphere:

A spherically symmetric charge distribution means the distribution of charge where the charge density ρ at any point depends only on the distance of the point from the centre and not on direction. Consider a total charge distributed uniformly throughout a sphere of radius r. Note that the sphere cannot be a conductor.

Case (i). When the point P lies outside the sphere

P is a point at a distance r from the centre O (Fig 1.5). We have to find the electric field E at P. Draw a concentric sphere (shown dotted) of radius OP with centre O. This is the Gaussian surface. At all points of this sphere, the magnitude of the electric field is the same and its direction is perpendicular to the surface. Angle between E and dS is zero

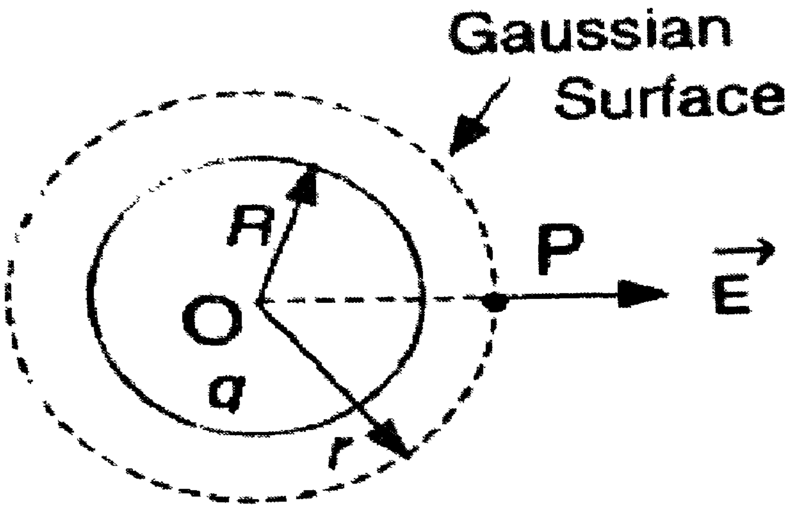


Fig1.5

The flux through this surface is given by

$$\oint E.dS = \oint E.dS = E(4\pi r^2)$$

By Gauss's law

$$E(4\pi r^2) = \frac{q}{\varepsilon_0}$$

or
$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

in vector form

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\,\hat{r}$$

Hence the electric field at an external point due to a uniformly charged sphere is the same as if the total charge is concentrated s its centre

Case (ii) when the point lies on the surface.

Here , r= R.

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$$

Case (iii) when the point lies inside the sphere

P' is a point inside the sphere (Fig.1.6). P' is at a distance r from the centre O. Draw a concentric sphere of radius r (r<R) with centre at O. This is the Gaussian surface.

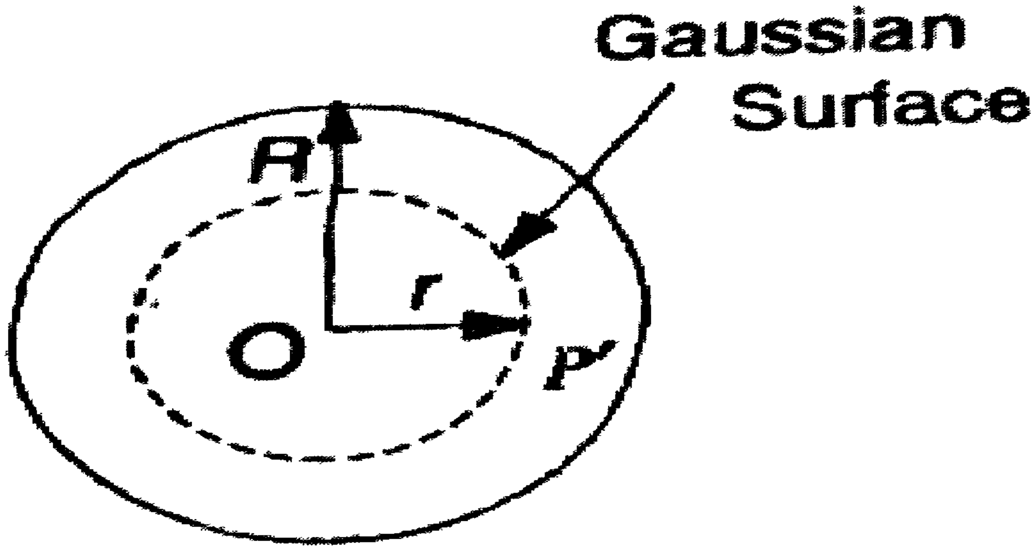


Fig 1.6

Total charge enclosed by the Gaussian surface.

$$q' = \frac{4}{3}\pi r^3 \rho$$

$$\frac{4}{3}\pi r^3 \frac{q}{\frac{4}{3}\pi R^3} = q\,\frac{r^3}{R^3}$$

Help ρ = charge density = charge per unit

Volume $= \frac{q}{\frac{4}{3}\pi R^3}$

The outward flux through the surface of the sphere of radius r is

$$\oint E.dS = E(4\pi r^2)$$

Applying Gauss' law

$$E(4\pi r^2) = \frac{q'}{\varepsilon_0} = \frac{q}{\varepsilon_0}\frac{r^3}{R^3}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^3}r$$

Thus $E \propto r$. At the centre of the sphere, E=0

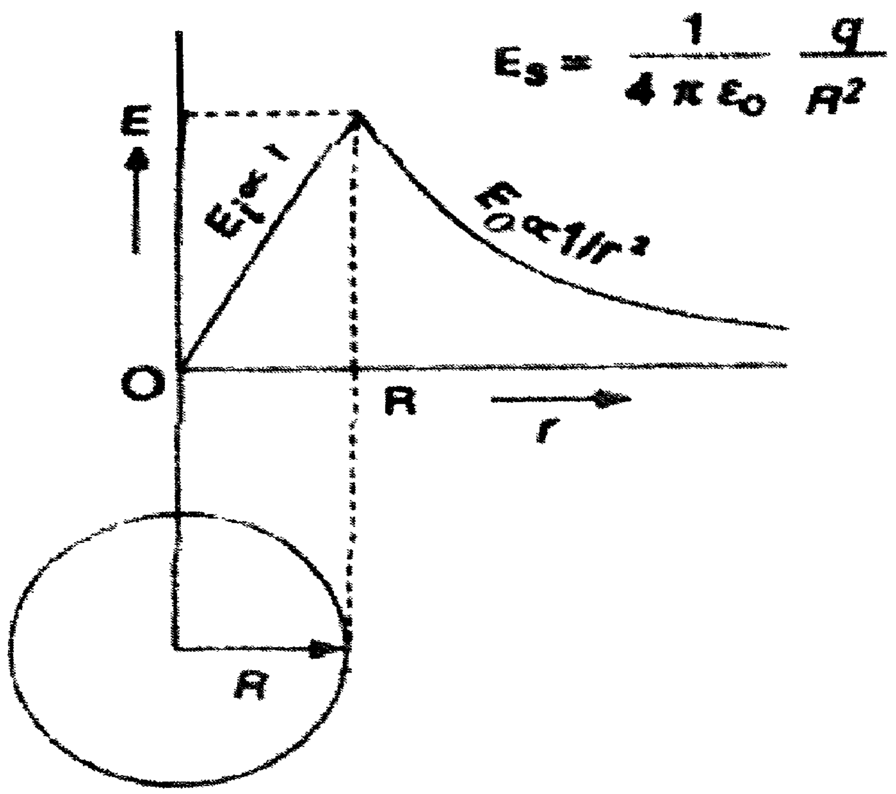


Fig1.7

Fig 1.7 shows the variation of electric field E as a function of radial distance r.

1.5.2 Electric field due to an isolated uniformly Charged Conducting Sphere

In an isolated charged spherical conductor any excess charge on it is distributed uniformly over its outer surface and there is no charge inside it. Hence this problem is the same as that of a charged spherical shell.

Case (i). At an external point

Consider a point P near but outside a uniformly charged sphere of radius R with a charge q (Fig 1.8). Let σ be the surface density of charge. Then $\sigma = \frac{q}{4\pi r^2}$. P is at distance r from the centre O.Draw a concentric sphere of radius OP with centre O. This is the Gaussian surface. Let E = electric field at any point on this sphere. At every point E is normal to the surface.

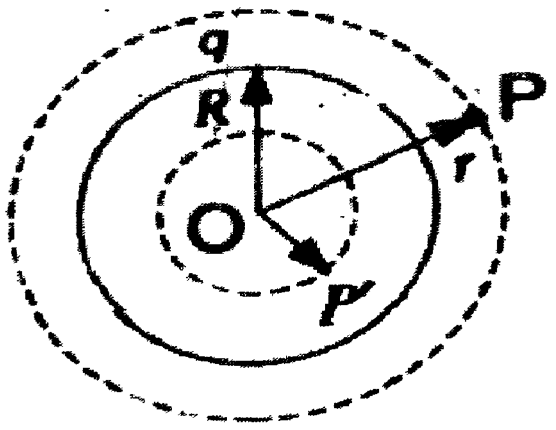


Fig 1.8

The flux through this surface is given by

$$\oint E.dS = \oint E.dS = E(4\pi r^2)$$

By Gauss's law

$$E(4\pi r^2) = \frac{q}{\varepsilon_0}$$

or.
$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

or
$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

The electric field is, therefore, the same as that due to the charge q situated at the centre of sphere. Therefore, for the points outside the sphere, the charges on the conducting sphere behave as if they were concentrated at the centre of the sphere.

Case (ii) at a point on the surface

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}\hat{r}$$

Case (iii) At a point inside

Let P' be an internal point. Through P' draw a concentric sphere. The charge inside this sphere is zero. Hence at all points inside the charged conducting sphere, electric field E=0.Fig 1.9 shows the variation of E with r.

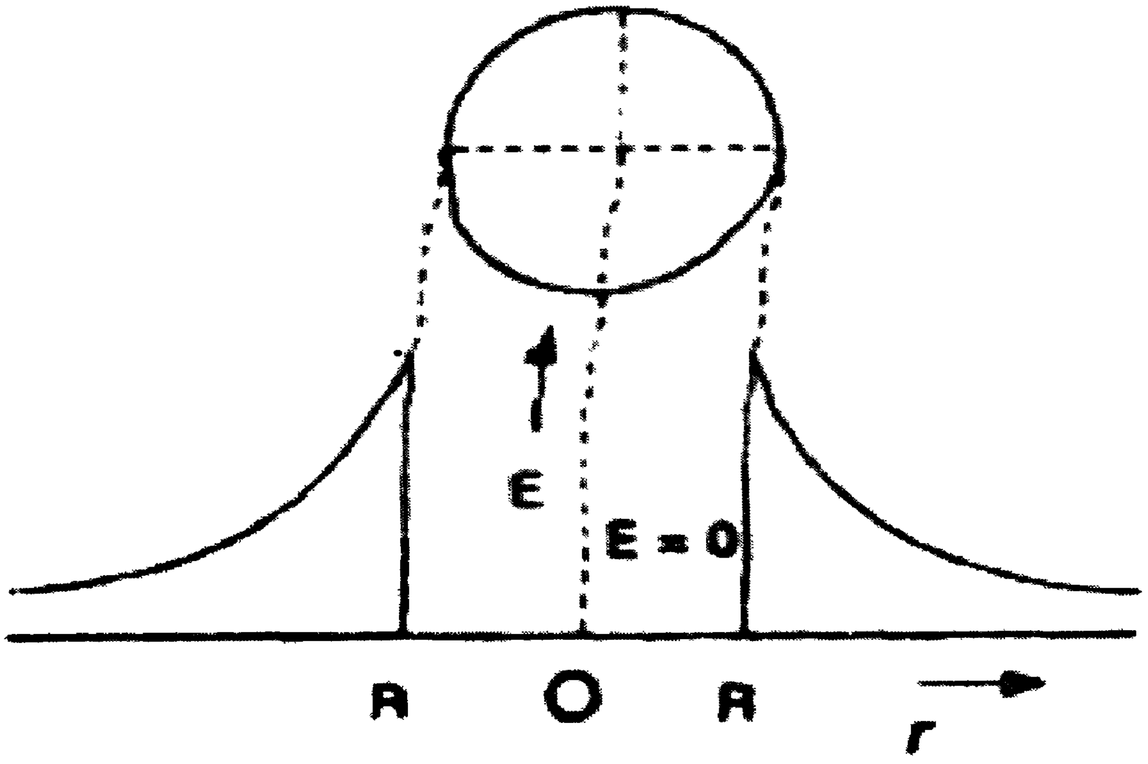


Fig 1.9

1.5.3 Electric Field due to an infinite plane sheet of charge

Consider a thin, non conducting infinite plane sheet of charge (Fig 1.10). Let this surface charge density (charge per unit surface area) be σ. Let P be a point at a distance r from the sheet. We want to calculate E at P. A convenient Gaussian surface is a "pill box" of a cross-sectional area A and height 2r, arranged to pierce the plane as shown. P' is symmetrical with P, on the other side of the sheet. From symmetry, E points at right angles to the end caps and away from the plane. Also its magnitude will be same at P and P'.

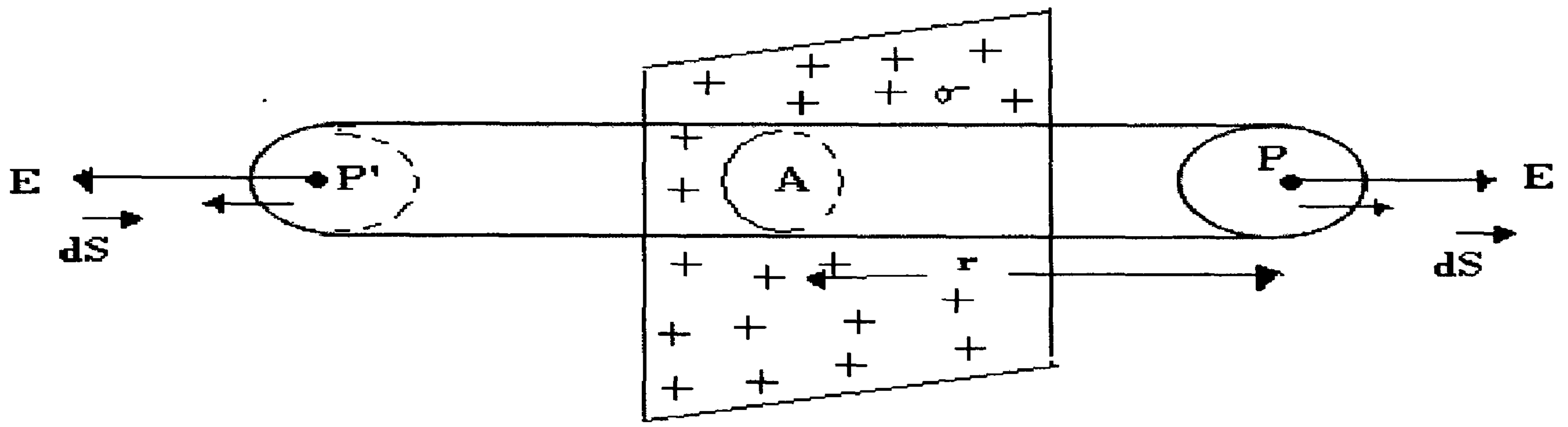


Fig 1.10

The flux through the two plane ends is

$$\varphi = \oint E.dS + \oint E.dS = \oint E\, dS + \oint E\, dS = EA + EA = 2EA.$$

The flux through the curved surface of the Gaussian cylinder is zero because E and dS are at right angles everywhere on the curve surface.

Total flux through the Gaussian cylinder $= \varphi = 2\ EA$.

Net charge enclosed by the Gaussian Cylinder = q = σA

By Gauss's law $\quad 2\ EA = \frac{\sigma A}{\varepsilon_0}$

$$E = \frac{\sigma}{2\varepsilon_0}$$

E is independent of the distance of the point from the sheet. E is the same for all point on each side of the plane.

1.6 Coulomb's Theorem

Statement: The electric field at any point near a charged conductor is $(1/\varepsilon_0)$ times the surface density of charge on the surface.

$$E = \frac{\sigma}{\varepsilon_0}$$

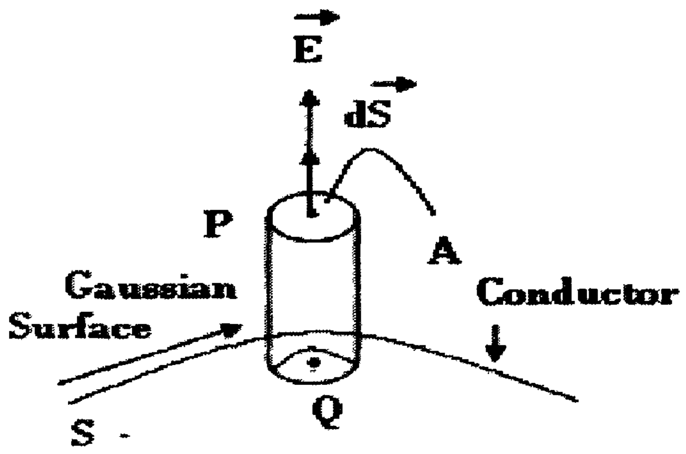


Fig 1.11

Proof: Consider a surface S of a charged conductor (Fig 1.11). Let σ be the surface density of charge. Consider two points P and Q one outside and the other inside the surface. Construct a Gaussian surface in the form of a cylinder, of area of cross-section A.

(i) E is zero everywhere inside the conductor. Hence the flux over the surface at Q=0

(ii) E is always normal to the charged conductor, and hence parallel to be curved surface. Therefore, the curved surface also does not contribute to the flux.

(iii)The only contribution to the flux is through the plane surface of area A at P which is outside the conductor.
Flux over the plane surface at P is

$$\varphi = \oint E.dS + \oint E\, dS = EA.$$

Net charge enclosed by the Gaussian surface = σA
From Gauss' law = EA = $\sigma A/\varepsilon_0$

$$E = \frac{\sigma}{\varepsilon_0}$$

Check your progress:
2. Give relation between Gauss theorem and Coulomb's law. Ans.:-- -- -- --

1.7 Mechanical Force on the surface of the charged conductor

Let σ is the surface density of charge on a conductor placed in vacuum. Consider small area dS on the surface (Fig 1.12). P is a point just outside the charged surface. The electric field E at P is the sum of two fields.

(i) E_1 is due to charges on the area dS and

(ii) E_2 is due to charges on the rest of the surface

E_1 and E_2 are in the same direction.

Total field at P is

$$E=E_1 + E_2 = \frac{\sigma}{\varepsilon_0} \text{ (by Coulomb's theorem) ...... (1)}$$

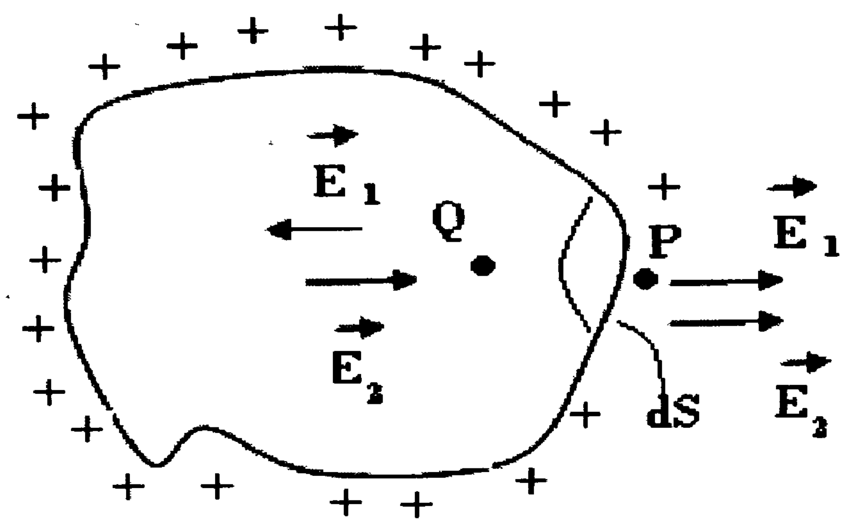


Fig 1.12

Consider a point Q just inside the conductor.

E_1 and E_2 are in opposite directions at this point

The field at Q is $E_1 - E_2 = 0$

or $E_2 = E_1$ ....(2)

or $E_1 = E_2 = \frac{E}{2} = \frac{\sigma}{2\varepsilon_0}$

$E_2 = \frac{\sigma}{2\varepsilon_0}$ ....(3)

Charge on the surface dS = $\sigma\, dS$

Therefore, the total mechanical force experienced by this charge due to the charge on the rest of the surface is

F=charge on dS x electric field E_2

$$= \sigma dS \times E_2 = \sigma dS \times \frac{\sigma}{2\varepsilon_0} \quad [from\ (3)\ E_2 = \frac{\sigma}{2\varepsilon_0}]$$

or $\quad F = \frac{\sigma^2\, dS}{2\varepsilon_0}$

Mechanical force experienced by unit area of the charged surface is called electrostatic pressure P.

$$P = \frac{\sigma^2\, dS/2\varepsilon_0}{dS} = \frac{\sigma^2}{2\varepsilon_0} \qquad ....(4)$$

But, $E = \frac{\sigma}{\varepsilon_0}$ or $\sigma = \varepsilon_0 E$

$$P = \frac{\varepsilon_0 E^2}{2} \qquad ...(5)$$

Its unit is Nm^{-2}. It is directed along the outward drawn normal to the surface. The surface is, therefore, under stress or outward electrical pressure.

1.8 Electric field and Electric potential

1.8.1 Electric field

If we place a test body carrying a positive electric charge q near a charged rod, an electrostatic force will act on it. We speak of an electric field in this region. We represent it by a vector E defined by

$$E = \frac{F}{q} \qquad ....(1)$$

Definition: Electric field at a point is defined as the force that acts on a unit positive charge placed at that point.

The direction of the vector E is that of the vector F.

The SI unit for the electric field is the Newton/coulomb (NC^{-1}) or volt/metre (Vm^{-1}).

Several other terms are commonly used, for example electric field strength, electric field intensity or intensity of the electric field. Eq.(1) may be written as

$$F = qE \qquad ....(2)$$

i.e., the force F exerted on a charge q at a point where the electric field is E equals the product of the electric field and the charge.

1.8.2 Electric Field due to a Point Charge

Let p be a point lying in vacuum at a distance r from a point charge q lying at O (Fig.1.13). Let a test charge q_0 be placed at P.

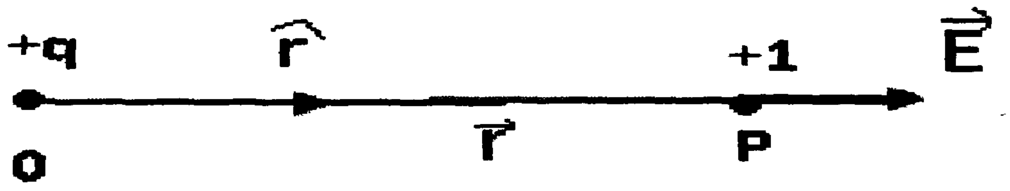


Fig 1.13 (a)

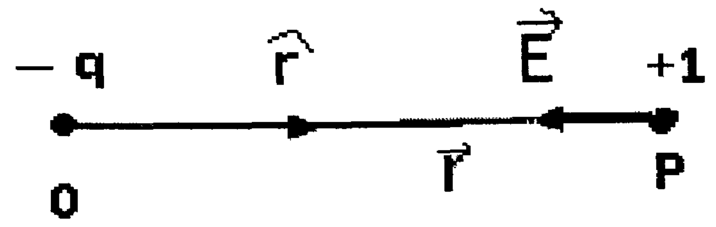


Fig 1.13 (b)

According to Coulomb's law, the force F acting on q_0 due to q is

$$F = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}\,\hat{r}$$

The electric field at the point P is, by definition, given by the force per unit test charge.

$$F = \frac{F}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\,\hat{r}$$

The direction of E is along the line joining O and P, pointing outward if q is positive [Fig 1.13(a)] and inward if q is negative [Fig 1.13.(b).]

1.8.3 Electric Potential

Consider an isolated point charge +q lying at O (Fig 1.14). A and B are two points in its electric field. Let W_{AB} be the work done by an external agent in moving a unit positive charge from A to B.

The potential difference between two points in an electric field is defined as the amount of work done in moving a unit positive charge from one point to the other against electrical forces.

$W_{AB} = V_B - V_A$

Here, V_A and V_B stand for the potentials at A and B.

The SI unit of potential difference is Volt. (Fig 1.14).

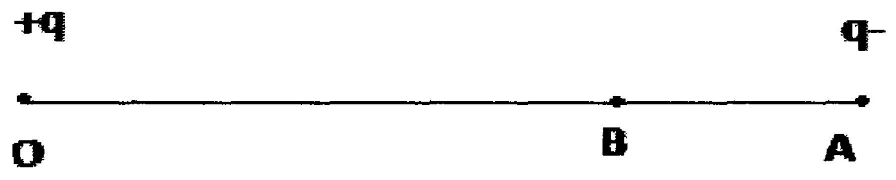


Fig 1.14

The potential difference between two points is 1 volt if 1 joule of work is done in moving 1 coulomb of charge from one point to the other against electrical forces.

Electric Potential: If A is at infinity, then $V_A = 0$

$$W = V_B$$

Here, W is the work done in moving a unit positive charge from infinity to the point B. V_B is the potential at B.

Definition: The electrical potential at a point is defined as the amount of work done in moving a unit positive charge from infinity to that point, without acceleration, against electrical forces.

The potential at a point near an isolated positive charge is positive.

The potential at a point near an isolated negative charge is negative.

1.8.4 Potential at a point due to a Point Charge:

Let + q be an isolated point-charge situated in air.

P is a point distant r from q (Fig 1.15)

The magnetic of the electric field at a distance r from the charge +q = E = $\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$

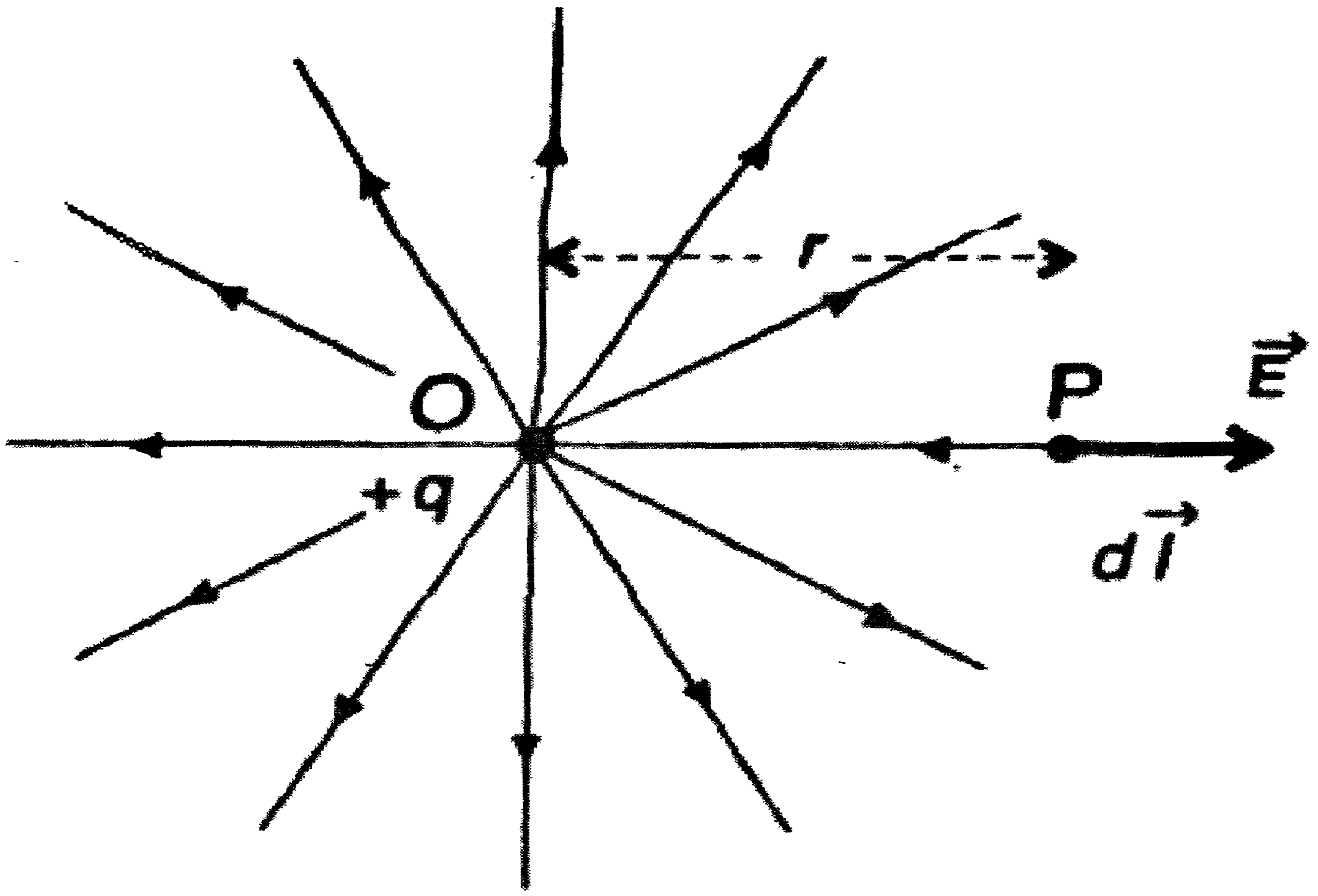


Fig 1.15

The potential at P is given by

$$V = -\int_{\infty}^{r} E.dl \qquad ....(1)$$

The displacement dl of the unit charge is directed towards the left. E is directed towards the right. Thus the angle between E and dl is 180°

$$E.dl = E\,dl\ \cos 180° = -E\,dl$$

Here, r is measured from the charge +q as origin. As we move a distance dl to the left, the value of r decreases. Thus dl = -dr

Equn. (1) becomes

$$V = -\int_{\infty}^{r} E.dl = -\int_{\infty}^{r} Edr = -\frac{q}{4\pi\varepsilon_0}\int_{\infty}^{r}\frac{dr}{r^2}$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} \qquad \ldots.(2)$$

1.8.5 Potential due to a Group of Point Charges

Potential is a scalar quantity. In order to find the potential at any point due to a group of point charges, we calculate the potential due to each individual charge, considering the other charges to be absent, and then add up these potentials algebraically.

Thus, if there are n point charges, the potential at a point due to them is

$$V = \frac{1}{4\pi\varepsilon_0} = \sum_{n}\frac{q_n}{r_n}$$

Here, q_n is the value of the nth charge and r_n is the distance of this charge from the point.

1.8.6 Relation between Electric Field and Electric Potential

We can calculate the electric field E if potential function V is known throughout a certain region of space. Consider two neighbouring points A (x,y,z) and B (x +dx, y+dy, z +dz) distance dl apart in the region (Fig 1.16).

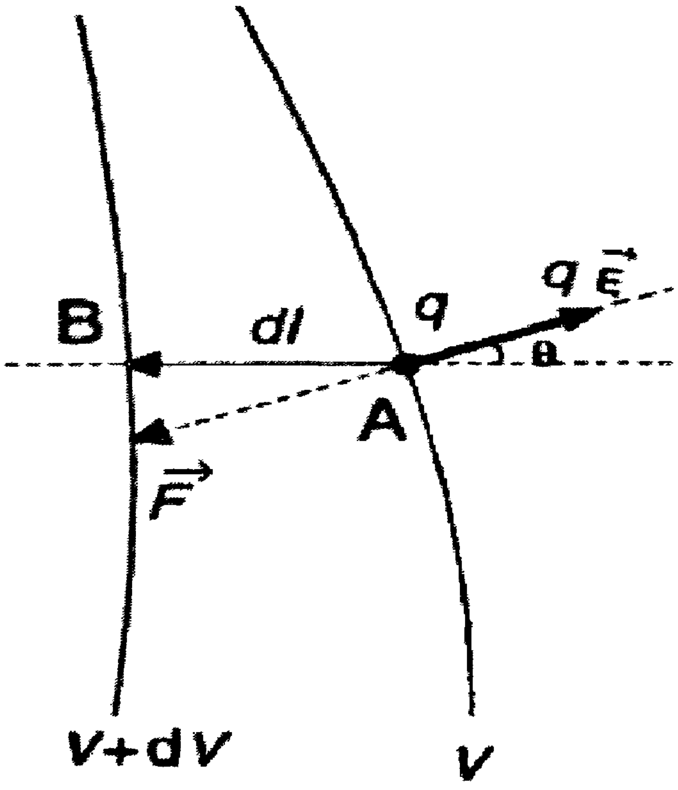


Fig1.16

Let the value of potential at A and B be V and V + dV respectively. Let dV be the change in potential in going from A to B. Then

$$dV = \frac{\partial V}{\partial x}ds + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz$$

$$= \left(\boldsymbol{i}\frac{\partial V}{\partial x} + \boldsymbol{j}\frac{\partial V}{\partial y} + \boldsymbol{k}\frac{\partial V}{\partial z}\right)(\boldsymbol{i}\, dx + \boldsymbol{j}\, dy + \boldsymbol{k} dz)$$

Here, $\boldsymbol{i}\, dx + \boldsymbol{j}\, dy + \boldsymbol{k} dz$ is the displacement vector d*l* between A and B.
Thus

$$dV = (grad\ V).\ dl \qquad ...(1)$$

The work done by the external agent in moving a test charge q from A to B along dI is

$$dW = F.dl\ = -qE.dl$$

or

$$\frac{dW}{q} = -E\ .dl$$

buy, by definition , $\frac{dW}{q}$ is the potential difference dV between the points A and B.
thus

$$dV = -E\ .dl$$

Comparing Eqs.(1) and (2),

$$E = -grad\ V = -\nabla V \qquad .....(3)$$

Thus the electric field at any point is the negative of the gradient of potential at the point. The minus sign indicates that E points in the direction of decreasing V.
Let E_x, E_y and E_z be the components of E along x,y and z axes. Then

$$E_x = -\frac{\partial V}{\partial x};\ E_y = -\frac{\partial V}{\partial y};\ E_z = -\frac{\partial V}{\partial z}$$

1.9 Electric field due to an electric dipole at a point on its axial line and equatorial line

1.9.1 Electric field due to an electric dipole at a point on its axial line

AB is an electric dipole of two point charges –q and +q separated by a small distance 2d (Fig 1.17) P is a point along the axial line of the dipole at a distance r from the midpoint O of the electric dipole.

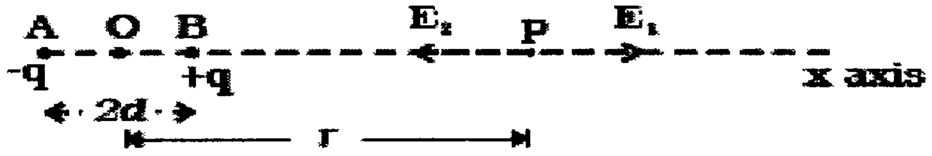


Fig1.17

The electric field at the point P due to +q placed at B is.

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-d)^2}\ (along\ BP)$$

The electric field at the point P due to -q placed at A is.

$$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r+d)^2}\ (along\ PA)$$

$$E_1\ and E_2\ act\ in\ opposite\ directins.$$

Therefore, the magnitude of resultant electric field (E) acts in the direction of the vector with a greater magnitude. The resultant electric field at P is

$$E = E_1 + (-E_2)$$

$$E = \left[\frac{1}{4\pi\varepsilon_0}\frac{q}{(r-d)^2} - \frac{1}{4\pi\varepsilon_0}\frac{q}{(r+d)^2}\right] \text{ along BP.}$$

$$E = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r-d)^2} - \frac{1}{(r+d)^2}\right] \text{ along BP.}$$

$$E = \frac{q}{4\pi\varepsilon_0}\left[\frac{4rd}{(r^2-d^2)^2}\right] \text{ along BP.}$$

If the point P is far away from the dipole, then $d<<r$

$$E = \frac{q}{4\pi\varepsilon_0}\frac{4rd}{r^4} = \frac{q}{4\pi\varepsilon_0}\frac{4d}{r^3}$$

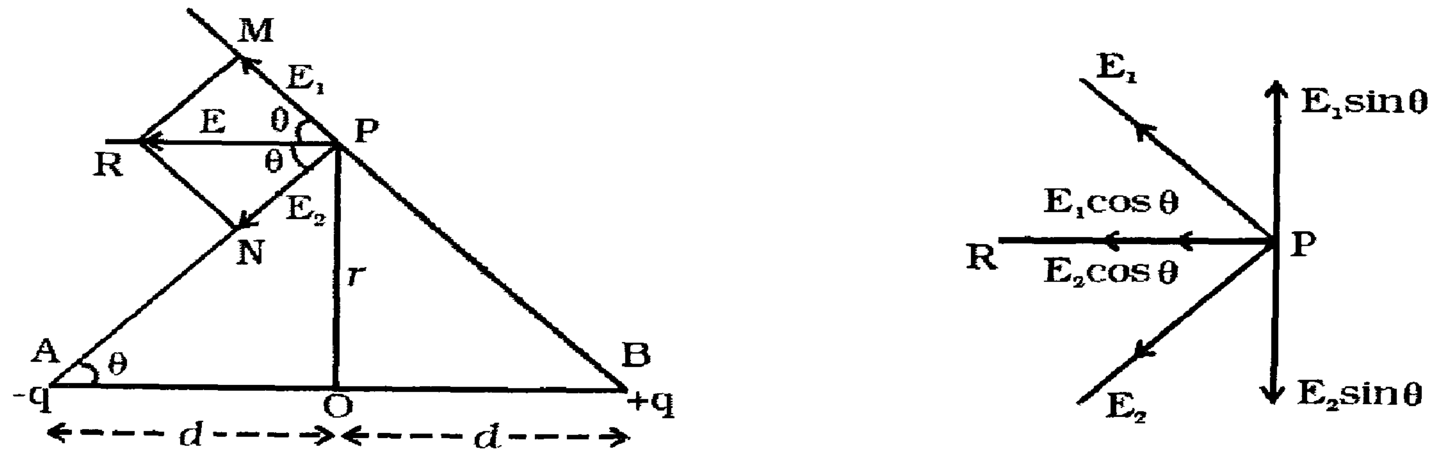
$$E = \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3} \quad \text{along BP.}$$

[Electric dipole moment p=q*2d]

E acts in the direction of dipole moment.

9.2 Electric Field due to an electric dipole at a point on the equatorial line

Consider an electric dipole AB. Let 2d be the dipole distance and p be the dipole moment. P is a point on the equatorial line at a distance r from the midpoint O of the dipole (Fig.1.18.a)



(a) Electric field at a point on equatorial line

(b) The components of the electric field

Fig 1.18

Electric field at a point P due to the charge +q of the dipole is

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{BP^2}\ (along\ BP)$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{(r+d)^2} \text{ along BP.} \quad (BP^2 = OP^2 + OB^2)$$

Electric field (E_2) at a point P due to the charge –q of the dipole is

$$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{AP^2}\ (along\ PA)$$

$$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r+d)^2}\ (along\ PA)$$

The magnitude of E_1 and E_2 are equal. Resolving E_1 and E_2 into their horizontal and vertical components (Fig 1.18.b), the vertical components $E_1 \sin\theta$ and $E_2 \sin\theta$ are equal and opposite, therefore they cancel each other.

The horizontal components $E_1 \cos\theta$ and $E_2 \cos\theta$ will get added along PR.

$E = E_1 \cos\theta + E_2 \cos\theta$(along PR)

$= 2 E_1 \cos\theta \quad (E_1 = E_2)$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r^2+d^2)} X \cos\theta$$

but $$\cos\theta = \frac{d}{\sqrt{r^2+d^2}}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r^2+d^2)} \text{ X } \frac{2d}{(r^2+d^2)^{3/2}} \qquad (p = q\ 2d)$$

For a dipole, d is very small when compared to r

$$E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}$$

The direction of E is along PR, parallel to the axis of the dipole and directed opposite to the direction of dipole moment.

1.10 Potential due to a charged conductor

Consider a hollow spherical conductor of radius R carrying a positive charge of σ coulombs/square metre. Let P be a point outside the spherical conductor at a distance r from the centre O of the sphere (Fig 1.19)

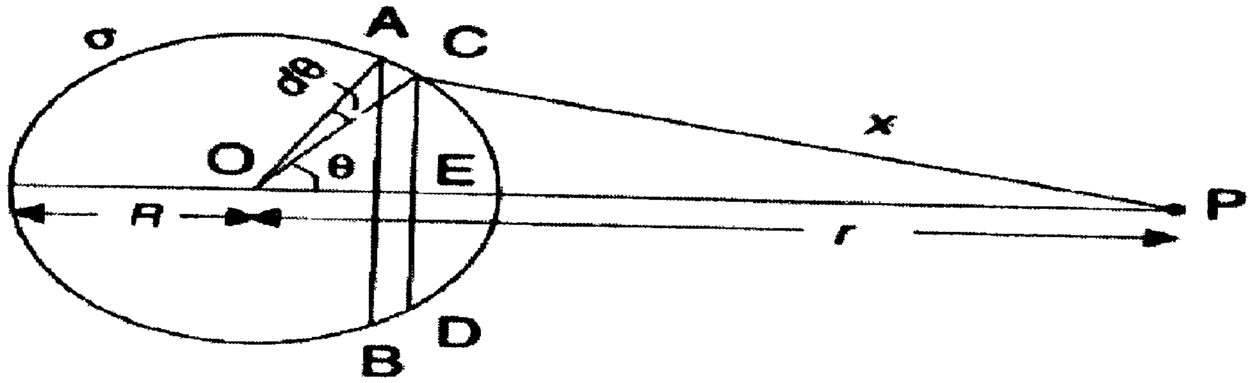


Fig1.19

Q coulombs is the positive charge distributed uniformly on the outer surface of the hollow sphere.
Surface density if charge = $\sigma = q/4\pi R^2$).

(i) Potential at an external point. P is a point at a distance r from the centre of the sphere. Draw two parallel planes AB and CED so as to form an annular ring.

$$CP = x, < COP = \theta, < COA = d\theta, CE = R\sin\theta, AC = Rd\theta$$

Area of the ring = (2 πR sin θ) $(Rd\theta) = 2\pi R^2 \sin\theta d\theta.$

Charge on the ring ABCD = $\sigma = (2\pi R^2 \sin\theta d\theta) = \frac{1}{2} q \sin\theta d\theta.$

Each point on this narrow ring is at the same distance x from P.

Potential at P due to the charge on the ring

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{(\frac{1}{2}q\sin\theta d\theta)}{x} \quad .....(1)$$

$$\text{From } \Delta\, COP, x^2 = R^2 + r^2 - 2Rr\cos\theta \quad ....(2)$$

Here, both x and θ are variables, differentiating.

2x dx = 2Rr sin θdθ

or $\quad \sin\theta d\theta = \frac{xdx}{Rr}$

Substituting in Eq.(1) we get

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{q\, dx}{2\, Rr}$$

The whole sphere is divided into such narrow rings.

Potential at P due to the whole sphere is given by

$$V = \int_{r-R}^{r+R} \frac{1}{4\pi\varepsilon_0}\frac{q\, dx}{2\, Rr} = \frac{1}{4\pi\varepsilon_0}\frac{q}{2\, Rr}\int_{r-R}^{r+R} dx$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} \quad ...(4)$$

Hence the potential at any point outside a charged sphere is the same as if the whole charge on the sphere is concentrated at its centre.

Electric Field: We know that E = - dV/dr

Electric Field outside the charged sphere $E_0 = \frac{d}{dx}\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{r}\right)$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

Its direction is along OP.

(ii) If P lies on the surface of the sphere, r = R

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R} \quad ....(5)$$

(iii) Potential at an internal point

If P lies inside, then the limits of integration are R – r and R + r

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{2Rr}\int_{r-R}^{r+R} dx$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R} \quad ....(6)$$

Thus the potential at all points inside a uniformly charged conducting sphere is the same.

Electric field inside the charged sphere = $E_1 = -\frac{d}{dx}\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{R}\right) = 0$

The variation of potential inside and outside the sphere is shown in Fig.1.20

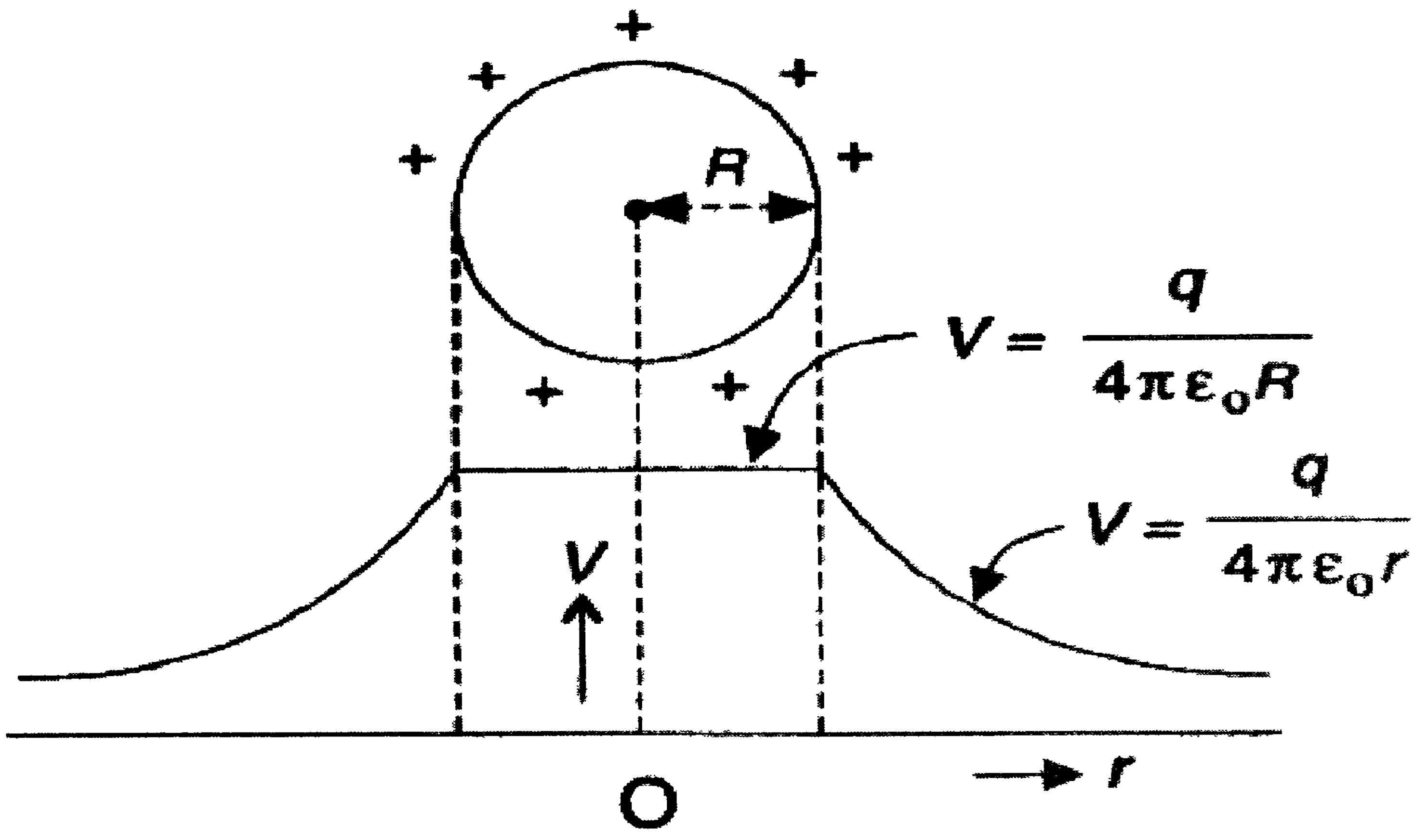


Fig1..20

Check your progress:
3. The electric field near the earth's surface is 300 volt/metre directed downwards. What is the surface density of charge on the earth's surface? Ans:-- -- -- --

1.11 Capacitance – Principle and Expressions for capacitance

1.11.1 Capacitance of a conductor

If a charge q is given to an isolated conductor, its voltage is increased by an amount V. For a given conductor, the ratio Q/V is independent of Q and depends only on the size and shape of the conductor. The ratio Q/V is called the capacitance of the conductor and is denoted by C.

Definition: The capacitance of a conductor is defined as the ratio of the charge given to the increase in the potential of the conductor.

The capacitance of a conductor is also defined as the amount of charge that should be given to it to increase its potential by unity. The unit of capacitance is farad. A conductor has a capacitance of one farad, if a charge of 1 coulomb given to it raises its potential by 1 volt. $1\mu F = 10^{-6}F$; $1pF = 10^{-12}$ F.

1.11.2 Principle of a capacitor

Suppose an insulated metallic plate A is given a positive charge Q and its potential is V (Fig 1.21). Its capacitance C = Q/V.

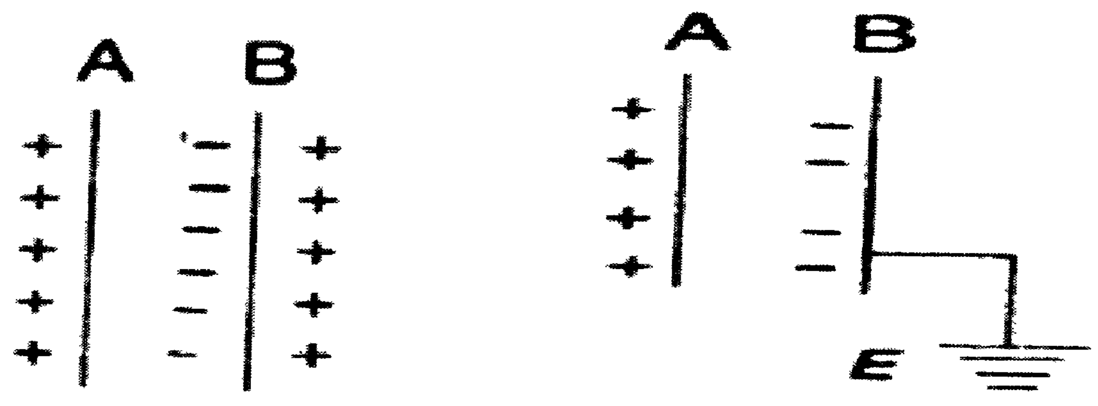


Fig 1.21 & 1.22

Let another insulated metal plate B be brought near A. Negative charge is induced on that side of B which is nearer to A. An equal positive charge is induced on the other side B. The negative charge on B decreases the potential charge on B decreases the potential of A. The positive charge on B increases the potential of A. But the negative charge on B is nearer to A than the positive charge on B. So the net effect is that the potential of A decreases. Thus the capacitance of A is increased.

The positive charge on B is neutralized by connecting the back side of B to earth (Fig 1.22). Then the potential of A decreases still further. Thus the capacitance of A is considerably increased.

A capacitor in general consists of two conductors, one positively charged and the other earthed. The conductors are called plates. The capacitance depends on the geometry of the conductors and the permittivity of the medium separating them. A capacitor is a device for storing charge.

Effect of a Dielectric: In actual capacitors, the region between its two conductors is filled with an insulator (or dielectric) say mica or oil. Faraday found that the capacitance of a capacitor increases if a dielectric is placed between the plates.

If C is the capacitance of a capacitor with vacuum and C' is its capacitance with dielectric, then the ratio $\frac{C'}{C} = \varepsilon_r$ is called the relative permittivity of the medium

$$C' = \varepsilon_r C$$

1.11.3 **Capacitance of a Spherical Capacitor (outer sphere earthed):**

Let A and B be two concentric metal spheres of radii a and b respectively with air as the intervening medium (Fig 1.23). The outer sphere B is earthed. A charge +q is given to the inner sphere. The induced charge on the inner surface of the outer sphere is –q. P is a point at a distance r from the common centre O.

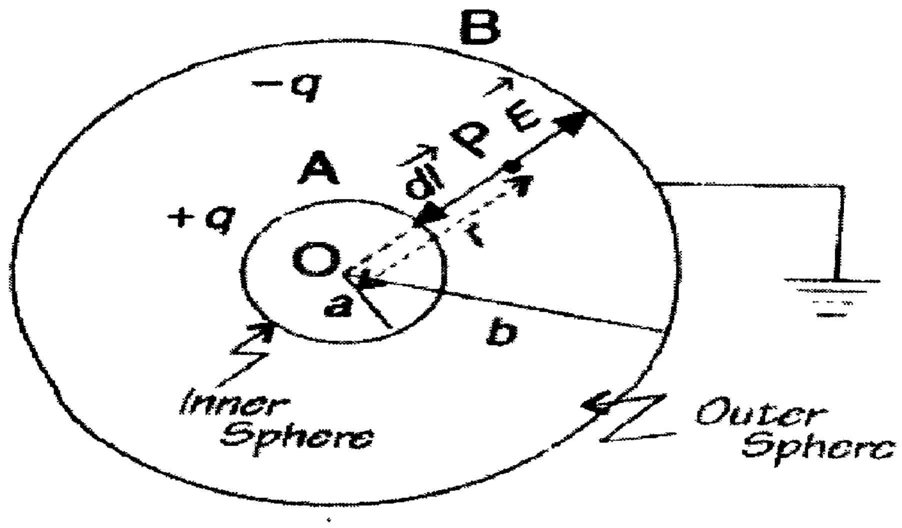


Fig1.23

Electric filed at P = $E = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r^2}\right)\hat{r}$ ....(1)

Where $\hat{r}$ is the unit vector along $\overrightarrow{OP}$.

The potential difference between the sphere A and B is given by

$$V = -\int_b^a E.dl \quad .....(2)$$

Here, dl is the differential vector displacement along a path from B to A.

But E.dl = E dl cos 180° = - E dl

Further, in moving a distance dl in the direction of motion, we are moving in the direction of r decreasing. So that dl=-dr. Hence,

$$\mathbf{E}.\, dl = E\, dr$$

Eq.(2) becomes $V = -\int_b^a E dr$

Putting the value of E from Eq.(1), we get

$$V = -\frac{q}{4\pi\varepsilon_0}\int_b^a \frac{dr}{r^2} = \frac{q}{4\pi\varepsilon_0}\left\{-\frac{1}{r}\right\}_b^a$$

$$= \frac{q}{4\pi\varepsilon_0}\left\{\frac{1}{a} - \frac{1}{b}\right\} = \frac{q}{4\pi\varepsilon_0}\frac{(b-a)}{ab}$$

Capacitance of the spherical capacitor

$$C = \frac{q}{V} = \frac{q}{\left(\frac{q}{4\pi\varepsilon_0}\right)\left(\frac{b-a}{ab}\right)} = 4\pi\varepsilon_0\frac{ab}{(b-a)} \quad \text{......(3)}$$

Note: Eq.(3) can be written in the form

$$C = 4\pi\varepsilon_0\frac{ab}{(b-a)} = \frac{4\pi\varepsilon_0}{\left(\frac{1}{a} - \frac{1}{b}\right)}$$

When b $\rightarrow \infty, C = 4\pi\varepsilon_0 a$

This is the capacitance of an isolated conducting sphere of radius a.

Capacitance of Spherical Capacitor (Inner Sphere Earthed)

A and B are two sphere of radii a and b (Fig 1.24). Suppose a charge +q is given to the outer sphere B. +q is distributed on its inner and outer surfaces by amounts $+q_1$ and $+q_2$ respectively, so that $q = q_1 + q_2$. The charge $+q_1$ on the inner surface of B induces a charge $-q_1$ (bound charge) on the outer surface of A and charge $+q_1$ on the inner surface of A. the charge $+q_1$ on the inner surface of A, being free, leaks to the earth.

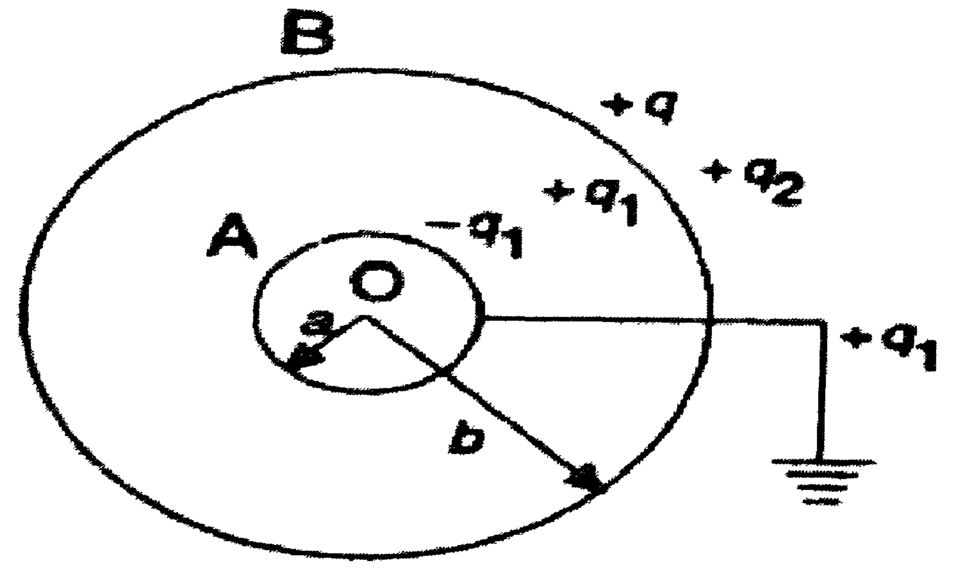


Fig1.24

The two spheres now behave as two capacitors connected in parallel.

(i) The inner sphere of radius a and the inner surface of outer sphere form a capacitor of capacitance

$$C_1 = \frac{4\pi\varepsilon_0 ab}{(b-a)} \quad \text{(if the dielectric is air)}$$

(ii) The outer surface of B and the earth form a capacitor of capacitance.

$$C_2 = 4\pi\varepsilon_0 b$$

Total capacitance $C = C_1 + C_2 = \frac{4\pi\varepsilon_0 ab}{(b-a)} + 4\pi\varepsilon_0 b$

$$C = \frac{4\pi\varepsilon_0 b^2}{(b-a)}$$

1.11.4 **Capacitance of a Cylindrical Capacitor**

Consider a cylindrical capacitor formed by two coaxial cylinder A and B of radii a and b respectively and each of length l. Air is the medium between A and B. The outer cylinder B is earthed (Fig 1.25)

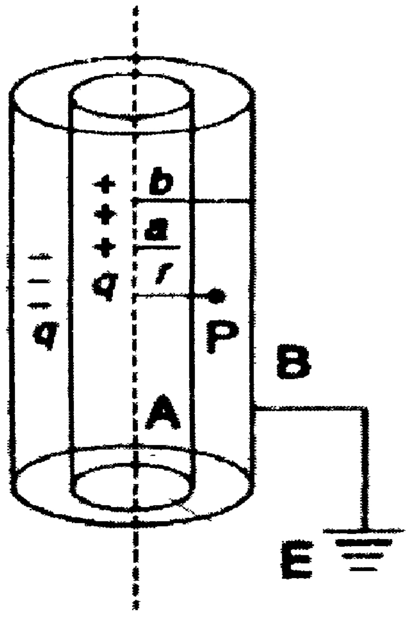


Fig1.25

If a charge +q is given to the inner cylinder, then an equal charge –q is induced on the inner surface of the outer cylinder and a charge +q on the outer surface of the outer cylinder. The charge +q induced on the outer surface of the outer cylinder flows to the earth.

The electric field at a point P in the space between the two cylinders at a distance r from the axis is

$$E = \frac{1}{2\pi\varepsilon_0 l}\frac{q}{r} \quad ....(1)$$

The potential difference V between the cylinder A and B is

$$V = -\int_b^a E.dl \quad .....(2)$$

Here , dl is the vector displacement along a path from B to A (Fig 1.26)

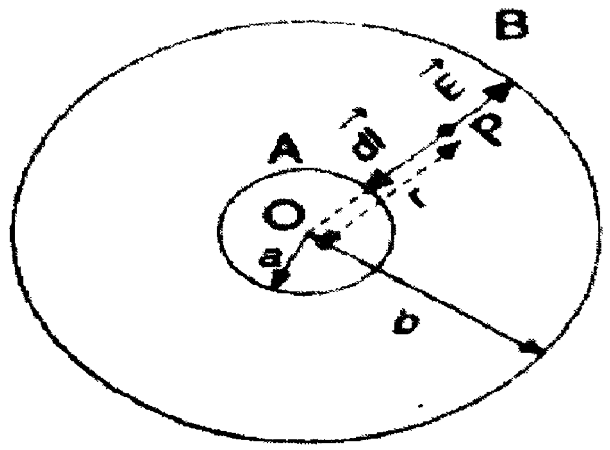


Fig1.26

Now , E is radially outward and dl is inward. Therefore

E.dl = E dl cos 180° = - E dl

As we move a distance dl from B to A, we move in the direction of decreasing r. So dl = -dr. Thus

E. dl = E dr

Eq(2) becomes $V = -\int_b^a Edr$

$$= -\frac{q}{2\pi\varepsilon_0 l}\int_b^a \frac{dr}{r} \qquad [from\ Eq.(1)]$$

$$= -\frac{q}{2\pi\varepsilon_0 l}\{log_e r\}_b{}^a = -\frac{q}{2\pi\varepsilon_0 l}\{log_e a - log_e b\}$$

$$= -\frac{q}{2\pi\varepsilon_0 l} log_e\left(\frac{b}{a}\right)$$

Hence the capacitance of the cylindrical capacitor is

$$C = \frac{q}{V} = \frac{2\pi\varepsilon_0 l}{log_e(\frac{b}{a})}$$

If the space between the two cylinders contains some medium of relative permitted ε_r the above expression becomes

$$C = \frac{2\pi\varepsilon_0\varepsilon_r l}{log_e(\frac{b}{a})}$$

Examples of Practical Cylindrical Capacitors:

The Co-axial cable consists of a cylindrical metal shield, a co-axial central conductor and an in interposed dielectric.

(i) A submarine cable consists of strands of copper separated from the surrounding water by a suitable insulating cable. It thus acts as a cylindrical capacitor. The copper strands form the inner cylinder. The surrounding water acts as the outer cylinder. The insulating casing acts as the dielectric.

1.11.5 Capacitance of Parallel Plate Capacitor

The parallel plate capacitor consists of two parallel metal plates each of area A and B separated by a distance d (Fig 1.27). The medium between the plates is air. A charge +q is given to the plate P. It induces a charge –q on the upper surface of the earthed plate Q. 'd' is kept small compared with the plate dimensions to enable us to ignore the fringing effects near the ends. Thus electric lines of force starting from plate P and ending at the plate Q are parallel to each other and perpendicular to the plates.

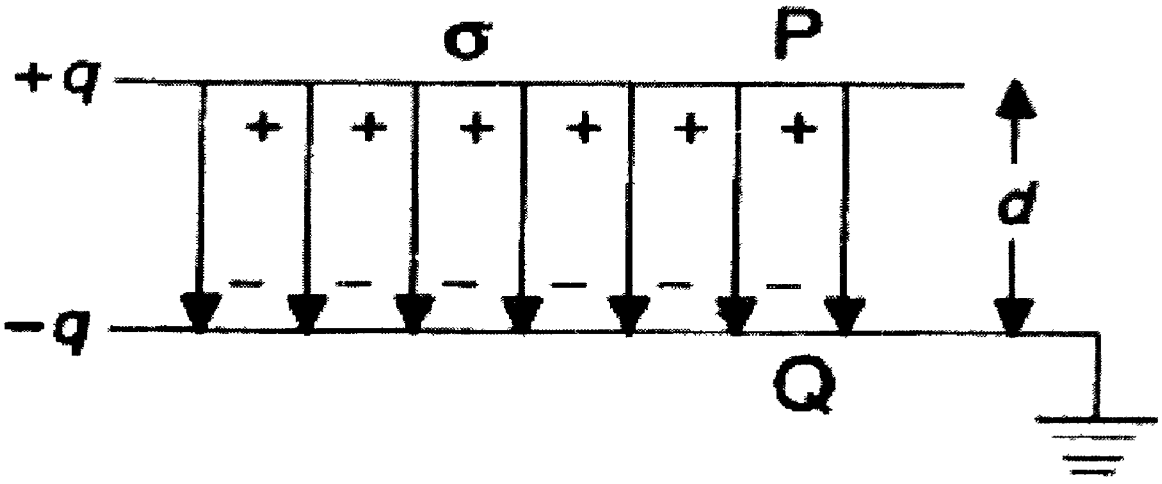


Fig1.27

By the application of Gauss's law

Electric field at a point between the two plates = $E = \frac{\sigma}{\varepsilon_0}$

Here, σ = surface density of charge = q/A

Potential difference between the plates P and Q is

$$V = \int_d^0 -E\,dr = \int_d^0 -\frac{\sigma}{\varepsilon_0}dr = \frac{\sigma d}{\varepsilon_0}$$

The capacitance of the parallel plate capacitor is

$$C = \frac{q}{V} = \frac{\sigma A}{(\sigma d/\varepsilon_0)} = \frac{\varepsilon_0 A}{d}$$

$$C = \frac{\varepsilon_0 A}{d}$$

Capacitance of Parallel Plate Capacitor Partly Filled with Dielectric Slab:

P and Q are the conducting plates of a parallel plate capacitor, each of area A placed at a distance, d apart. Suppose a dielectric slab, of thickness t and relative permittivity ε_r is introduced between the plates (Fig 1.28) P is given a positive charge so that the surface charge density on it is σ.

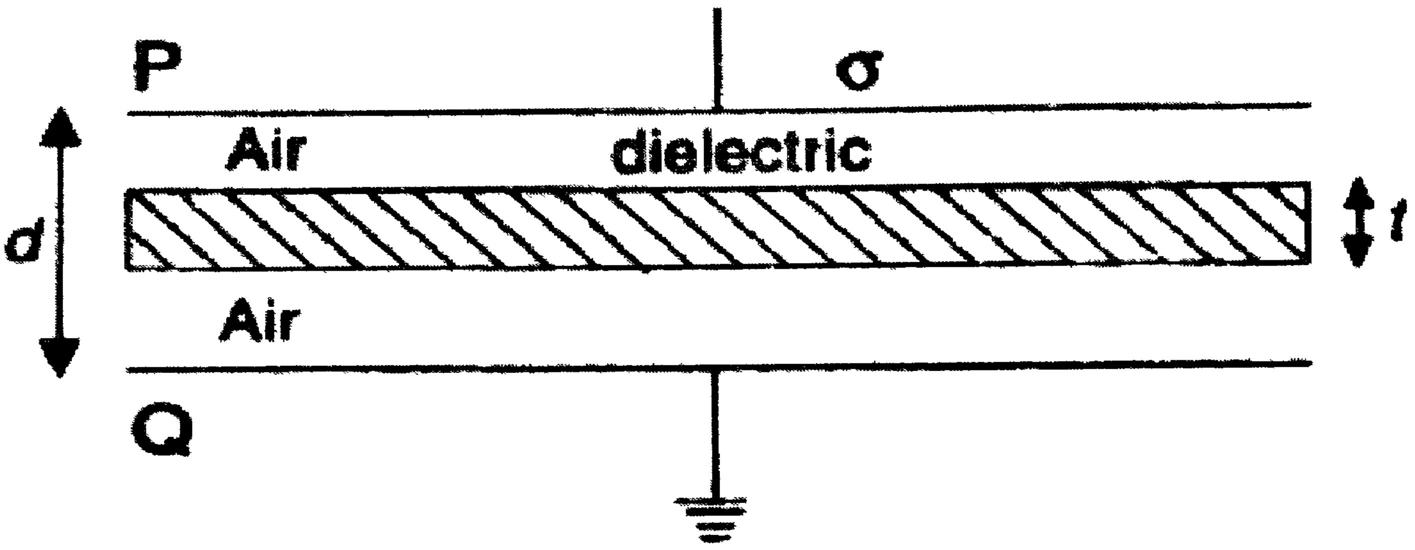


Fig1.28

Thickness of dielectric slab = t

Thickness of air portion = (d-t)

Electric field at any point in the air space between the plates $=E = \frac{\sigma}{\varepsilon_0}$

Electric field at any point in the dielectric slab $= E = \frac{\sigma}{\varepsilon_r \varepsilon_0}$

The potential difference V between the plates is the work done in carrying a unit positive charge from one plate to other in the field E over a length (d-t) and in the field E over a length t. Thus

$$V = E(d-t) + E't = \frac{\sigma}{\varepsilon_0}(d-t) + \frac{\sigma t}{\varepsilon_r \varepsilon_0}$$

$$= \frac{\sigma}{\varepsilon_0}\left[(d-t) + \frac{t}{\varepsilon_r}\right]$$

The charge on the plate P = q = σA

Hence the capacitance of the capacitor is

$$C = \frac{q}{V} = \frac{\sigma A}{\frac{\sigma}{\varepsilon_0}\left[(d-t) + \frac{t}{\varepsilon_r}\right]} = \frac{\varepsilon_0 A}{(d-t) + \frac{t}{\varepsilon_r}}$$

1.12 **Energy of Capacitor**

Let q' be the charge and V' the potential difference established between the plates of the capacitors at any instant during the process of charging. If an additional charge dq' is given to the plates, the work done by the battery is given by

$$dW = V'dq' = \left(\frac{q'}{C'}\right) dq'$$

$$(V' = \frac{q'}{C})$$

Total work done to charge a capacitor to a charge q is

$$W = \int dW = \int_0^q \frac{q'}{C'} dq' = \frac{1}{2}\frac{q^2}{C}$$

This work done is stored as electrostatic potential energy in the capacitor

$U = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}CV^2$ (q = CV)

This energy can be recovered if the capacitor is allowed to discharge.

Energy Density :

Consider a parallel plate capacitor of area A and plate separation d.

Energy of the capacitor = $U = \frac{1}{2}CV^2 = \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)V^2$

Volume of the space between the plates = Ad

Energy density u is the potential energy per unit volume.

$$u = \frac{U}{Ad} = \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}V^2\right) \times \frac{1}{Ad} = \frac{1}{2}\varepsilon_0 \left(\frac{V}{d}\right)^2$$

$u = \frac{1}{2}\varepsilon_0 E^2$ (V/d = E)

Thus we can associate an electrostatic energy density $u = \frac{1}{2}\varepsilon_0 E^2$ with energy point in space where an electric field E exists.

1.13 Loss of Energy on Sharing of charges between Two Capacitors :

Consider two capacitors of capacitances C_1 and C_2 charged to potentials V_1 and V_2 (Fig 1.29).

Fig1.29

When they are joined by a wire, they attain a common potential V.

$$V = \frac{Total\ charge}{Total\ Capacitance} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

Total energy of the two capacitors before contact

$$U_1 = \frac{1}{2}C_1{V_1}^2 + \frac{1}{2}C_2{V_2}^2 \quad \ldots(1)$$

Total energy of the two capacitors after contact

$$U_2 = \frac{1}{2}(C_1 + C_2)V_1{}^2 = \frac{1}{2}(C_1 + C_2)\left[\frac{C_1V_1 + C_2V_2}{C_1 + C_2}\right]^2$$

$$= \frac{1}{2}\frac{(C_1V_1+C_2V_2)^2}{(C_1+C_2)} \quad \ldots\ldots\ldots\ldots(2)$$

Loss of energy due to contact,

$$U_1 - U_2 = \frac{1}{2}C_1V_1{}^2 + \frac{1}{2}C_2V_2{}^2 - \frac{1}{2}\frac{(C_1V_1+C_2V_2)^2}{(C_1+C_2)}$$

$$= \frac{1}{2(C_1 + C_2)}\left[(C_1 + C_2)(C_1V_1{}^2 + C_2V_2{}^2) - (C_1V_1 + C_2V_2)^2\right]$$

$$= \frac{1}{2(C_1 + C_2)}[C_1V_1{}^2 + C_1C_2V_2{}^2 + C_1C_2V_1{}^2 + C_2V_2{}^2 - C_1{}^2V_1{}^2$$
$$- C_2V_2{}^2 - 2C_1C_2V_1V_2]$$

$$= \frac{C_1 + C_2}{2(C_1 + C_2)}[V_1{}^2 + V_2{}^2 - 2V_1V_2]$$

$$= \frac{C_1 + C_2}{2(C_1 + C_2)}(V_1V_2)^2$$

Since $(V_1V_2)^2$ is always positive, U_2 must be less than U_1. Hence there is loss of energy on sharing the changes. The loss of energy appears partly as heat in the connecting wire and partly as light and sound if sparking occurs.

1.14 Types of capacitors

(a) Leyden jar: This capacitor has played an important historical role in electricity. It consists of a glass jar which is covered, both inside and outside including the bottom, with tin foil (Fig 1.30). The inside coating is connected through a chain to a rod which projects outside the jar. The rod is fitted with a knob at its upper end. The tin foil coatings act as the parallel plates of a capacitor with glass acting as the separating dielectric.

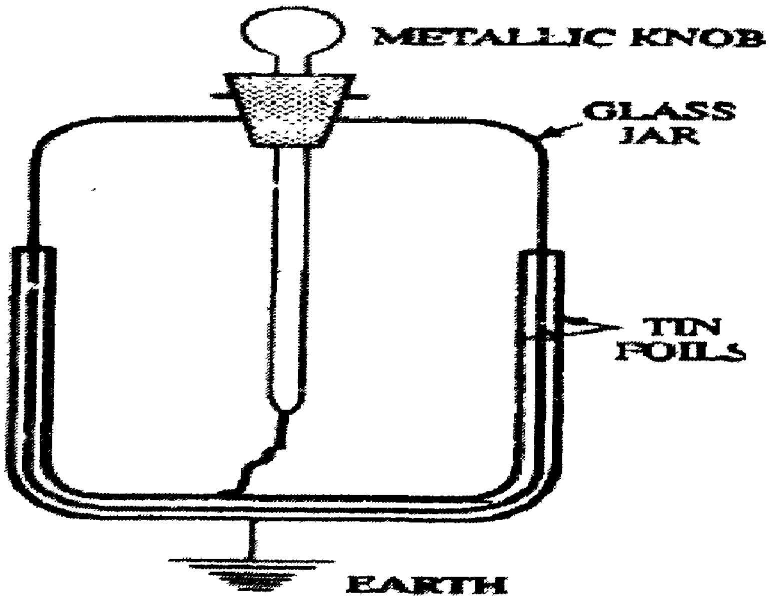


Fig1.30

This is similar to a parallel place capacitor

The capacitance of the Leyden jar $C = \frac{\varepsilon_r \varepsilon_0 A}{t}$

ε_r is the relative permittivity of glass

A is the overlapping area of tin foils between the inner and the outer coatings

t is the thickness of glass

(b) Guard Ring capacitor. In parallel plate capacitor, the electric field between the plates is not uniform near the edges. This is called the "edge effect" of "fringing". The express C= ε_0 A/d is only approximate. This is avoided by using a guard ring. The circular insulated plate P is surrounded by a circular coplanar ring G. the inner diameter of G is slightly larger than the diameter of P (Fig 1.31)

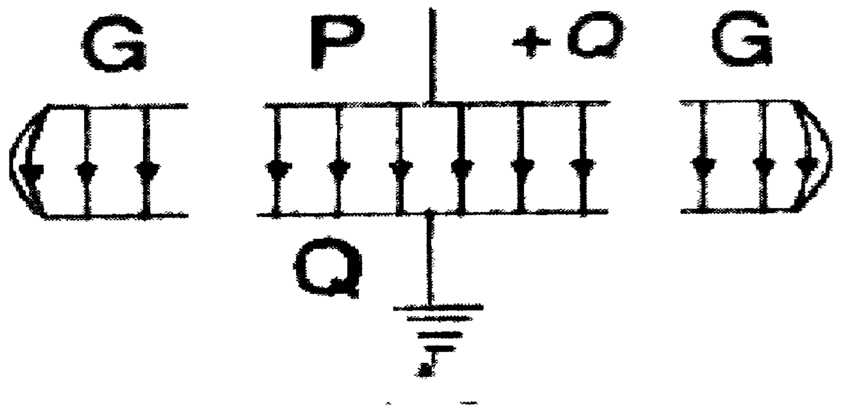


Fig1.31

The air gap between P and G is very small. The diameter of the plate Q is equal to the outer diameter of G. The field between P and Q is uniform throughout the common area between them. The irregularity in the field occurs at the outer edge of the guard ring.

The effective area of the plate = A' = Area of the plate P + $\frac{1}{2}$Area of the circular air gap between P and G.

C= ε_0 A'/d

This is used as an absolute standard of capacitance.

(c)Mica Capacitor

A schematic diagram of a multiple capacitor is shown in Fig 1.32 consists of a number of parallel plate capacitors in parallel with the alternate metallic foils fixed to one end each. Mica is used as the dielectric. The capacitance of such a system is C =n ε_r ε_0 A/d where n is the number of capacitors grouped in parallel, A is the surface area of the plate and d is the thickness of each mica sheet.

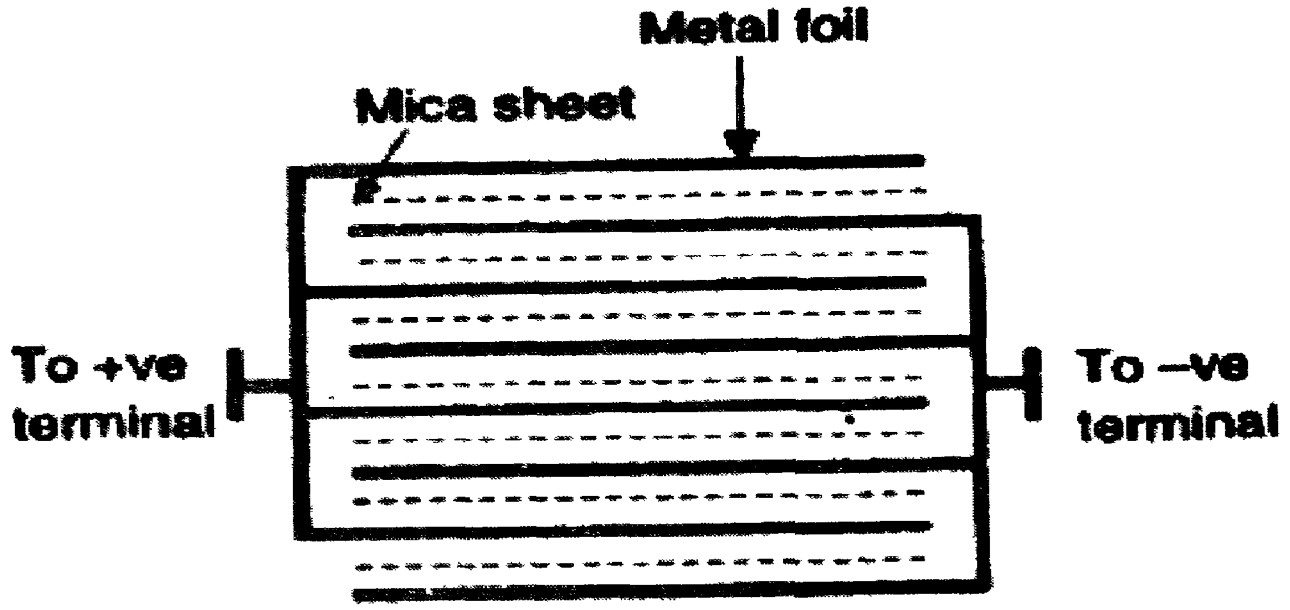


Fig1.32

(d) Electrolytic Capacitor

It consists of two aluminium electrodes A and C dipped in a solution of ammonium borate (fig 1.33).

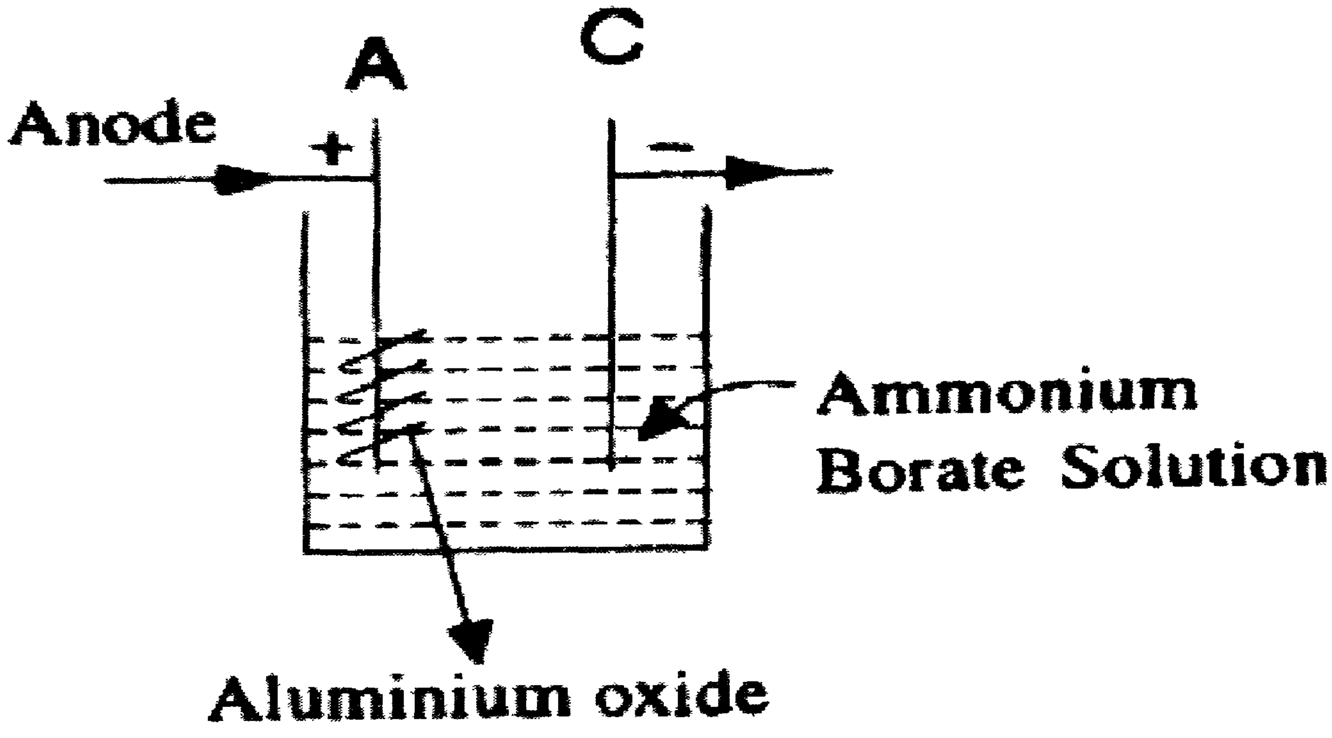


Fig1.33

On passing a direct current, a very thin film of aluminium oxide is formed on the anode. This film is an insulator. The arrangement can now be used as a capacitor with the anode as one plate, the solution as the other plate, and the aluminium oxide film as dielectric. Since the dielectric layer is very thin, the capacitance of this arrangement is very large. This capacitor must be placed only in a D.C circuit. It cannot be used in an A.C circuit.

(e) Variable Air Capacitor

In consists of two sets of metal plates, one fixed and the other movable (Fig 1.34).

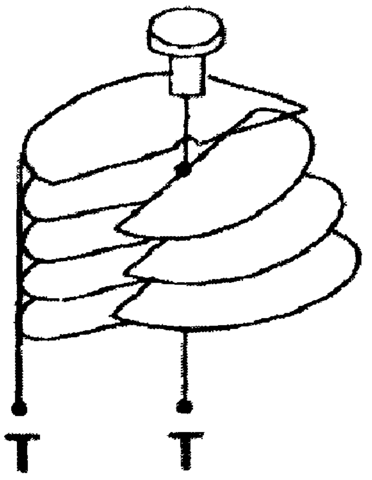


Fig1.34

The fixed set is semi-circular in shape. The movable set is like a can and rotated with knobs. All the fixed plates are connected to one terminal. All the movable plates are connected to another terminal. Air is the dielectric. By rotating the knobs, the area of overlap between the two sets of plates is changed. Thus the capacitance of the capacitor changes. These capacitors are widely used in the tuning circuits of radio receivers.

Uses of Capacitors:

(i) They are used in the ignition system of automobile engines for eliminating sparking

(ii) They are used in radio circuits for tuning, to reduce voltage fluctuations in power supplies, and to increase the efficiency of alternating current power transmission.

(iii) They are used to generate and detect electromagnetic oscillations of high frequency.

(iv) They serve as useful devices for storing electric energy.

Check your progress:
4. What is the value of capacitance of parallel plate capacitor when the distance of separating between the plates is halved? Ans:--

Check your progress:
5. What will be happened to the capacitance of a capacitor when air in a capacitor is replaced by a medium of dielectric constant K.? Ans:--- --- ---

1.15 Let us Sum up

- **Coulomb's Law:** The force between two point charges is directly proportional to the product of the charges, and inversely proportional to the square of the distance between them.

$$F_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\,\hat{r}$$

- **Gauss's Law:** The total electric flux (θ) of the electric field E over any closed surface is equal to ($1/\varepsilon_0$) 1 times the total net charge (q) enclosed by the surface.

$$\varphi = \oint E.dS = \frac{q}{\varepsilon_0}$$

- **Coulomb's Theorem:** The electric field at any point near a charged conductor is ($1/\varepsilon_0$) times the surface density of charge on the surface.
- **Mechanical Force experienced by unit Area of a Charged Conductor**

$$P = \frac{\varepsilon_0 E^2}{2}$$

- **Electric field:** Electric field at a point is defined as the force that acts on a unit positive charge placed at that point.

$$E = \frac{F}{q}$$

- **Electric field due to an electric dipole at a point on its axial line**

$$E = \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}$$

- **Electric field due to an electric dipole at a point on its equatorial line**

$$E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}$$

- **Electric Potential Difference:** The potential difference between two points in an electric field is defined as the amount of work done in moving a unit positive charge from one point to the other against electrical forces.

- **Capacitance :** The capacitance of a conductor is defined as the ratio of the charge given to the increase in the potential of the conductor. The unit of capacitance is farad.
- **Capacitance of a Spherical Capacitor:**

$$C = 4\pi\varepsilon_0 \frac{ab}{(b-a)} = \frac{4\pi\varepsilon_0}{\left(\frac{1}{a}-\frac{1}{b}\right)} \quad \text{(Outer Sphere Earthed)}$$

$$C = \frac{4\pi\varepsilon_0 b^2}{(b-a)} \quad \text{(Inner Sphere Earthed)}$$

- **Capacitance of a Cylindrical Capacitor:**

$$C = \frac{2\pi\varepsilon_0\varepsilon_r l}{log_e(\frac{b}{a})}$$

- **Capacitance of Parallel Plate Capacitor:**

$$C = \frac{\varepsilon_0 A}{d}$$

- **Types of capacitors:**
  - Leyden jar.
  - Guard Ring capacitor.
  - Mica Capacitor.
  - Electrolytic Capacitor.
  - Variable Air Capacitor.

1.16 **Unit – end exercises :**

1. Define Electric field and Electric field intensity.
2. Derive an expression for the electric field due to a point charge
3. State Gauss Theorem. Obtain an expression for the electric flux in Electrostatics.
4. State and explain Gauss' Law.
5. State and explain Gauss' Law in electrostatics.
6. State and prove Gauss' Theorem in electrostatics.
7. Explain the mechanical force on the surface of a charged conductor.
8. Define electric potential and its unit.
9. Define the terms 'potential' and 'equipotential surface'.
10. Discuss the relation between electric intensity and electric potential.
11. Derive the relation between the electric field at a point and the electric potential.
12. Explain the principle of a capacitor.
13. Define Capacity and Farad.
14. Applying Gauss Theorem find Electric field at points inside and outside a uniformly charged non-conducting sphere.

15. State and prove Coulomb's Theorem in Electrostatics.
16. Derive an expression for the Mechanical force per unit area of a charged conductor.
17. Calculate the Excess of pressure inside a charged soap bubble.
18. Derive the expression for the capacity of a spherical condenser.
19. Derive expression for the capacity of a cylindrical condenser.

1.16 Problems for discussion:

1. Four grams of gold are beaten into a thin leaf of area 1 m^2. A small piece is cut of it and placed on a conductor. Calculate the surface density of charge required by the conductor so that the gold leaf is just lifted up.
2. A sphere diameter 0.05m is charged to a potential of 1000 volts. Calculate the outward-pull per unit area
3. A spark passes in air when the potential gradient at the surface of a charged conductor is 3×10^6 volts/m. What must be the radius of an insulated metal sphere which can be charged to a potential of 3×10^6 volt before sparking into air?
4. Calculate the electric potential energy of an electron- proton system of an atom. The radius of the orbit of the electron is 21.216×10^{-11} m. The charge on the electron is 1.6×10^{-19} C. (Given $\frac{1}{4\pi\varepsilon_0} = 9 \times \frac{10^9 Nm^2}{C^2}$)
5. Calculate the energy stored in a capacitor of capacitance 0.5 microfarad if it is charged to a potential of 200 volts.
6. Find the capacitance of parallel plate capacitor consisting of two parallel plates of area 0.054 m^2 each and placed 10^{-3} apart in free space.
7. The radii of the inner and outer spheres of a spherical capacitor are 4×10^{-2} metre and 6×10^{-2} metre. If the dielectric medium between the plates is air, calculate the capacitance for the spherical capacitor if the outer spheres is earthed and the inner sphere is positively charged.
8. A sphere of 10cm diameter is suspended within a hollow sphere of 12 cm diameter. If the inner sphere be charged to a potential of 15000 volt and the outer sphere be earthed, find the charge on the inner sphere.
9. A cable has a wire of radius 1 mm and it is surrounded by a thin metallic sheet of radius 6 mm. The space between the cable and the sheet is filled with a material of dielectric constant 2.05. what is the capacitance of 8 m length cable?

10. A cable consisting of a wire 3mm in diameter and insulated with 3 mm of gutta-percha ($\varepsilon_r = 4.26$) is placed in water. Calculate the capacitance of 1 km length of the cable?

1.17 Answers to check your progress and problems for discussion

Check your progress:

1. $F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}$

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 (2r)^2} = \frac{F}{4}$$

2. Gauss' theorem is a converse to Coulomb's law. From Coulomb's law, we find E from known source of charge. From Gauss theorem, we can find the source of charge, if we know E.

3. $E = \frac{\sigma}{\varepsilon_0}$ Here, E = 300 Vm^{-1} , $\sigma = ?$

$$\sigma = E\varepsilon_0 = 300\ X\ (8.84\ X\ 10^{-12}) = 2.6562\ X\ 10^{-9} Cm^{-2}$$

4. $C = \frac{\varepsilon_r\varepsilon_0 A}{d} = \frac{\varepsilon_r\varepsilon_0 A}{d/2} = 2c$

5. $\acute{C} = \varepsilon_r C$

$\acute{C} = KC$

The capacitance increased by K.

Problems for discussion:

1. Let σ = charge density on the surface of the conductor.
The upward force per m^2 due to charge = $\sigma^2/(2\varepsilon_0)$
Let m = mass per unit area of the gold foil
Downward force on the gold foil due to its weight = mg

When the gold foil is just lifted up $\frac{\sigma^2}{(2\varepsilon_0)} = mg$

$$\sigma = \sqrt{2\varepsilon_0 mg}$$

$$= \sqrt{2\ X\ (8.85\ X 10^{-12})(4\ X\ 10^{-3})9.8}$$

$$= 0.833\ X 10^{-6} Cm^{-2}$$

2. Charge on the sphere is

$$q = 4\pi\varepsilon_0 Vr = 4\pi(8.85\ X 10^{-12})1000\ X\ 0.025$$

$= 2.78\ X10^{-9}C.$

Surface charge density $= \sigma = \frac{q}{4\pi r^2} = \frac{2.78\ X10^{-9}}{4\pi\ (0.025)^2} = 3.54\ X10^{-7}Cm^{-2}$

Outward pull per unit area is

$$p = \frac{\sigma^2}{2\varepsilon_0} = \frac{(3.54\ X10^{-7})^2}{2\ X\ (8.85\ X10^{-12})} = 7.08\ X\ 10^{-3}Nm^{-2}$$

3. Here, potential gradient $= \frac{dV}{dt} = 3\ X\ 10^6 V/m$

or $dV = 3 \times 10^6\ dr$

or $V = 3 \times 10^6\ r$

But $v = 3 \times 10^6\ volt$

$3 \times 10^6\ r = 3\ x\ 10^6$

$r = 1m$

4. Here, $q_1 = 1.6 \times 10^{-19}$ C
$q_2 = -1.6 \times 10^{-19}$C
$r = 21.16 \times 10^{-11}$ m
P.E. $= \frac{q_1 q_2}{4\pi\varepsilon_0 r} = -\frac{(1.6\ X10^{-19})^2 \times (9\ X\ 10^9)}{(21.16 \times 10^{-11})} = -1.09 \times 10^{-18} J$

5. Energy stored $= \frac{1}{2}CV^2 = \frac{1}{2}(0.5 \times 10^{-6})(200)^2 = 0.01J$

6. $\varepsilon_0 = 8.85 \times 10^{-12}\ Fm^{-1}$

$$A = 4 \times 10^{-2}\ m^{-1}$$
$$d = 10^3 m$$

Therefore

$$C = \frac{8.85 \times 10^{-12} \times 4 \times 10^{-2}}{10^{-3}} = 3.54\ 10^{-10}F$$

Here C is the charge that raises the potential by unity or the charge holding capacity.

7. $C = \frac{4\pi\varepsilon_0 X\ ab}{(b-a)} = \frac{1}{9\times10^9} X \frac{4\ X\ 10^{-2}\ X\ 6\ X\ 10^{-2}}{2\ X\ 10^{-2}} = 1.33 \times 10^{-11}\ farad$

8. $C = \frac{4\pi\varepsilon_0 X\ ab}{(b-a)} = \left(\frac{1}{9\ X10^9}\right)\frac{0.05\ X\ 0.06}{0.01} = 3.333 \times 10^{-11}\ F$

Charge on the inner sphere = q = CV

$= (3.333 \text{ x } 10^{-11}) \text{ x } 15000 = 5 \text{ x } 10^{-7}C.$

9. Capacitance of cylindrical capacitor

$$C = \frac{2\pi\varepsilon_r\varepsilon_0 l}{log_e(\frac{b}{a})} = \frac{2\pi \ X\ 2.05\ X\ (8.854\ X10^{-12})X\ 8000}{2.3026\ X\ log_{10}(\frac{0.0065}{0.001})} = 0.5092 \text{ x } 10^{-6}\ F$$

10. The capacitance of cylindrical capacitor is given by

$$C = \frac{2\pi\varepsilon_r\varepsilon_0 l}{log_e(\frac{b}{a})}$$

Here , a= $\frac{3}{2}\ mm = 0.0015m$

$$b = \frac{3+6}{2}mm = 0.0045m;$$

$$\varepsilon_r = 4.26;\ l = 1km = 10^3 m$$

$$C = \frac{2 \text{ x } 3.14 \text{ x } 4.26 \text{ x } (8.854 \text{ x}10^{-12})\text{x } 10^3}{2.3026\ log_{10}\left(\frac{0.0045}{0.0015}\right)} = 2.156 \text{ x } 10^{-7}\ F$$

$$C = 0.2156\mu F.$$

1.18 Suggested Readings

1. Electricity and Magnetism - S Mahajan And A A Rangwala .Tata McGrew Hill
2. Electricity and Magnetism - Dr.K.K.Tewari S.Chand & Co,2002.
3. Electricity and Magnetism with Electronics -D.N.Vasudeva S.Chand & Co, 2002.
4. Electricity and Magnetism - Narayanamoorthy, Nagarathinam. 2nd Revised Edition

Unit II: Kirchoff's Laws

Structure

2.1. Introduction

2.2. Objectives

2.3. Kirchoff's laws

3.3.1. Application to wheatstone's bridge

2.3.2. Sensitiveness of bridge

2.4. Carey Foster's bridge

2.5. Theory – Principle of potentiometer

2.6. Determination of internal resisitance of the cell using potentiometer

2.7. Measurement of potential and Calibration of voltmeter (low Range)

2.7.1. Calibration of Voltmeter (High Range)

2.7.2. Measurement of Current and Calibration of ammeter

2.8. Seeback effect

2.9. Measurement of Thermo EMF using Potentiometer

2.10. Peltier effect – Peltier coefficient

2.11. Thomson coefficient

2.12. Thermoelectric power

2.13 Summary

2.14 Unit –end exercises

2.15 Point for discussion

2.16 Answers to check your progress

2.1. Introduction

Metals contain free electrons which can move through the metal under the action of electric force. Electric current in a conductor is the movement of these free electrons from one part to the other and is similar to the flow of water in a tube. As a difference of pressure is necessary at the two ends of a tube to make the water flow through it, similarly electrical pressure is necessary across the two points on the conductor for the electric current to flow through it.

In metallic conductors, the positive charge cannot move, the entire transfer of charge is only due to the movement of negatively charged particles, *i.e.*, electrons. The conventional current is in the opposite direction to the direction of movement of the electrons.

The atoms are firmly fixed in a metal and there is enough space in the metal for the movement of free electrons. An extremely small potential difference across any two points of a conductor is sufficient enough to make these electrons flow in the metal. In liquids and gases, the positively charged particles actually move in the direction of the conventional current and the negatively charged particles move in the opposite direction to the conventional current. Thus, in a metallic conductor the electric current that flows from the positive point to the negative point is in the opposite direction to the actual movement of the electrons and no positive charge actually moves from the positive to the negative point.

If two conductors charged to different potentials are connected by a conducting wire, neutralisation of charge takes place through the wire from the conductor at the higher potential to the conductor at the lower potential. This flow of charge constitutes electric current and the current stops as soon as the potentials of the conductors become equal. If the potential difference between the conductors can be maintained by some means, there will be continuous flow of electric current through the conducting wire. This branch of electricity is called current electricity.

2.2. Objectives

After going through this unit, you will be able to:

- understand the basic concepts of current, current density and drift velocity.
- state Kirchhoff's law and its application to Wheat stone's bridge.
- derive the condition for the sensitivity of the bridge.
- describe the construction working and determination of the resistance.

- outline the features of See back, Peltier and Thomson's effect.
- state the laws of thermo emf.
- understand the importance of thermoelectric power.

Current and Current density

Electric current:

In an isolated metallic conductor, the free electrons present in it are in random motion like the molecules of a gas. So the net rate at which electrons pass through any hypothetical plane is zero. If the ends of the conductor are connected to a battery, an electric field E will be set up at every point within the conductor. This field acts on the electrons and gives them a resultant motion in the direction – **E**. If a charge q passes through any cross-section of the conductor in time t, then the current I is defined by

$$I=q/t$$

When q is in coulomb and t in second, then I is in ampere. If the rate of flow of charge with time is not constant, the current varies with time. Then at any instant current is defined as

$$I=dq/dt$$

So current is defined as the net charge flowing across the area per unit time. Current is a scalar quantity. Conventionally the direction of electric current is taken along the direction of motion of positive charges. When the current is caused by the motion of electrons, the direction of current is opposite to the direction of electron flow.

Current density:

Current is a macroscopic quantity. The related microscopic quantity is the current density J. The current density at any point is defined as the quantity of charge passing per second through a unit area taken perpendicular to the direction of the flow of charge at that point.

It is vector quantity and is characteristic of the given point inside a conductor. If the current is flowing uniformly across a conductor of cross-sectional are A, the magnitude of the current density for all points on that cross section is given by

$$J = \frac{q/t}{A} = \frac{i}{A}$$

The unit of current density if A/m^2

Current I is related to the current density J by

$$I = \int J.dS$$

where dS in an element of area and the integral is taken over any surface cutting across the conductor. From this equation we may define the current I as the flux of the current density vector J through a given area.

2.3 Kirchhoff's Laws

Law 1. In any network of conductors, the algebraic sum of the currents meeting at any point is zero i.e. $\Sigma I = 0$

In the above equation, current flowing into a junction is regarded as positive, while the current flowing out o the junction is regarded as negative. In Fig.2.1 is a junction. According to sign convention, I_1, I_2, I_5 are positive and I_3, I_4, I_6, are negative. Then,

$$I_1 + I_2 - I_3 - I_4 + I_5 - I_6 = 0$$

i.e. $$I_1 + I_2 + I_5 = I_3 + I_4 + I_6$$

Thus the sum of currents entering the point o is equal to the sum of currents leaving it. In other words Kirchhoff's first law states, "When steady current flows in an electric circuit, there is no accumulation of charge at any point or junction".

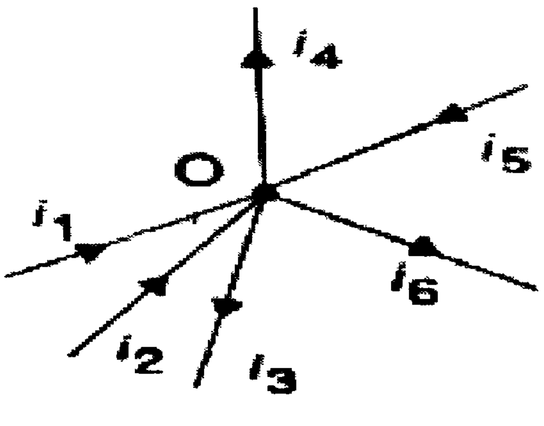


Fig 2.1

Law 2. The algebraic sum of the products of the current and resistance in any closed loop of a circuit is equal to the algebraic sum of electromotive forces (e.m.f.s) acting in that loop. i.e., Σ IR= ΣE.

Sign Convention. (i) A product of current and resistance is taken as positive when we traverse in the direction of the current.

(ii) the emf is taken as positive when we traverse from the negative to the positive electrode of the cell through the electrolyte.

Let i_1 and i_2 be the currents through R_1 and R_2 (Fig.2.2) applying first law to the junction a, the current through R is $i_1 + i_2$. Then by applying the second law for the closed loop ZAYZ.

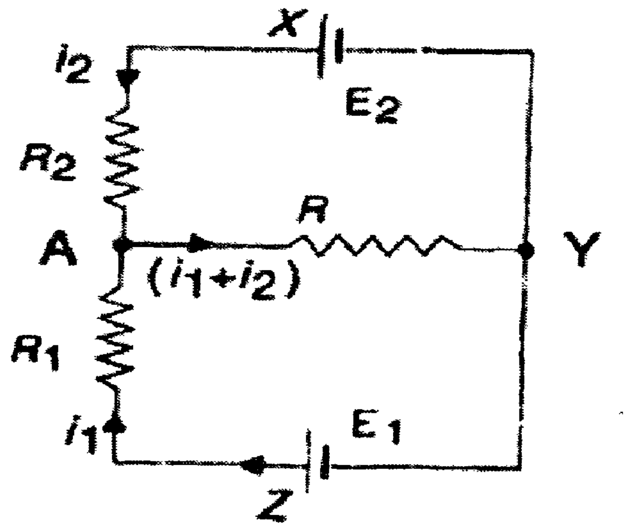


Fig 2.2

$$i_1R_1 + (i_1 + i_2)R = E_1$$

Similarly, for the closed loop XAYX,

$$i_2R_2 + (i_1 + i_2)R = E_2$$

These 'Loop equations' enable us to obtain the values of currents in different parts of the circuit in terms of R, R_1, R_2, E_1 *and* E_2

2.3.1 Application of Kirchhoff's laws to Wheat stone's bridge

Four resistances P, Q, S and R form a closed network ABCE (Fig 2.3) A cell of emf E is connected between A and C. A galvanometer of resistance G is connected between B and D.

Let I be the current along EA, I_1 along AB and I_g along BD. By Kirchhoff's first law, the current along the various branches will be as shown an expression for the current through the galvanometer i_g Kirchhoff's voltage law (KVL)

Applying KVL to the closed mesh ABDA.

$$I_1P + I_gG - (I - I_1)R = 0$$

or $$(P+R)\, I_1 + GI_g = IR \text{............}(1)$$

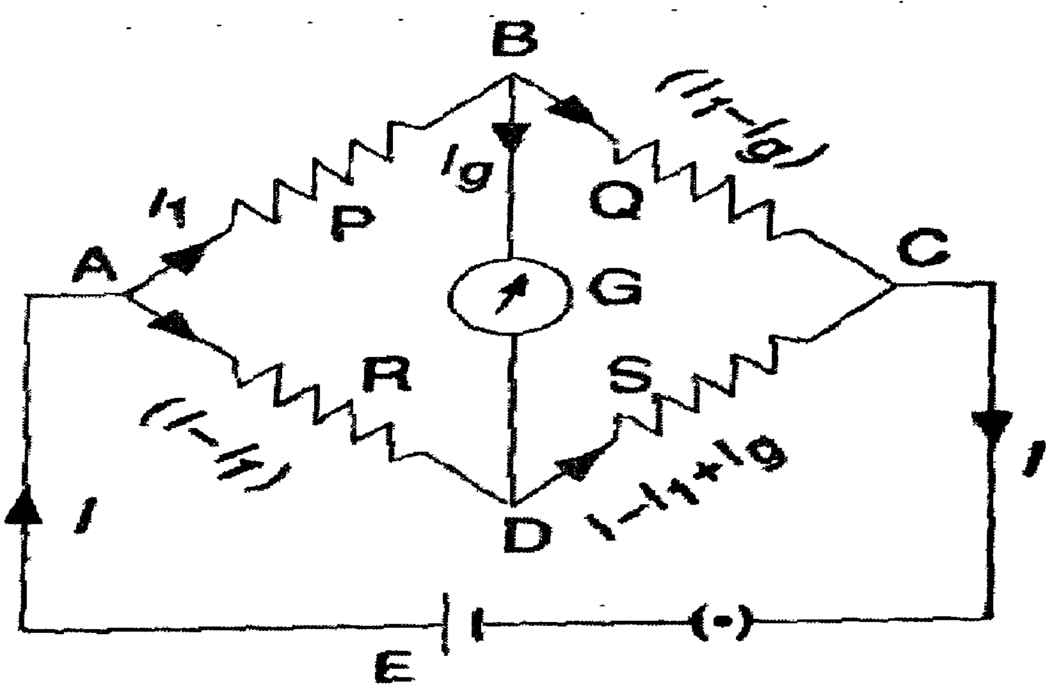


Fig 2.3

Applying KVL to the closed mesh BCDB,

$$(I_1 + I_g)Q - (I - I_1 + I_g)S - I_gG = 0$$

(or) $$(Q+S)\, I_1 - (Q+S+G)\, I_g = IS \text{.............}(2)$$

Multiplying Eq.(1) by (Q+S),

$$(P+R)(Q+S)\, I_1 + G(Q + S)I_g = R(Q + S)I \text{...........}(3)$$

Multiplying Eq. (2) by (P+R)

$$(P+R)(Q+S)\, I_1 - (P + R)(Q + S + G)I_g = S(P + R)I \text{......}(4)$$

Subtracting Eq.(4) from Eq.(3)

$$[G(Q+S)+(P+R)(Q+S+G)\, I_g = [R(Q + S) - S(P + R)]I$$

$$I_g = \frac{(QR-PS)I}{[G(P+Q+R+S)+(P+R)(Q+S)]} \text{..................}(5)$$

Condition for balance is $I_g = 0$ $i.e., (QR - PS)I = 0.$ Since I has a finite values,

QR-PS=0

Thus the condition for the balance of the bridge is

$$\frac{P}{Q}=\frac{R}{S} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

2.3.2 Sensitivity of Wheat stone's bridge

Fig 2.4 represents a Whetstone's bridge ABCD. The unknown resistance r is connected in the arm CD. Let the resistances in the arms AB, BC and AD be nmr_1, mr_1 and nr_1 respectively. Let G be the galvanometer resistance. Let I be the current through the unknown resistance r, I_g the current through the galvanometer and I_1 the current through resistance nmr_1. Then by Kirchhoff's I law, the currents through mr_1 and nr_1 are respectively (I_1+I_g) and $(I+I_g)$.

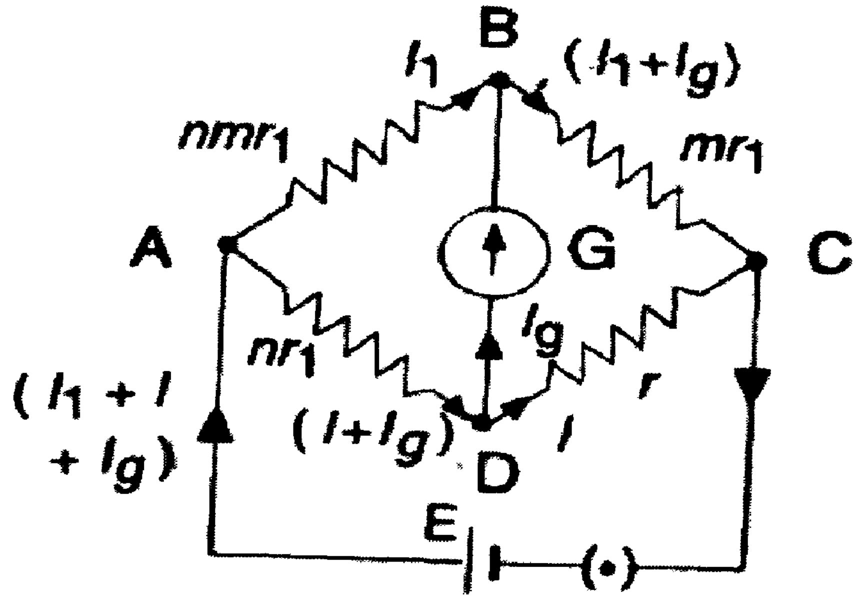


Fig 2.4

When r = r_1, the bridge is balanced and I_g =0. Therefore, the difference $(r-r_1)$ is a measure of the want of balance. The bridge will be most sensitive, if the galvanometer current I_g is large, even when $(r-r_1)$ is extremely small.

Applying KVL to the loops ABDA and BCDB respectively.

$$nmr_1I_1-GI_g-nr_1(I+I_g)=0\ldots\ldots\ldots(1)$$

$$mr_1(I+I_g)-rI+GI_g=0\ldots\ldots(2)$$

multiplying Eq.(2) by n, we get

$$nmr_1I_1+nmr_1 I_g-nrI+nGI_g=0\ldots\ldots\ldots(3)$$

subtracting Eq.(1) from Eq.(3), we get

$$G(1+n)\ I_g+nr_1\ (1+m)\ I_g-n(r-r_1)I=0$$

or $\frac{I_g}{I} = \frac{n(r-r_1)}{G(1+n)+ nr_1(1+m)}$

or $\frac{I_g}{I} = \frac{n(r-r_1)}{(1+1/n)G+ r_1(1+m)}$ ....(4)

For a given ant of balance $(r - r_1)$ the value of I_g will be large when,

(i) I, the current through the unknown resistance is large

(ii) G, the galvanometer resistance is small

(iii) N, the ration $\frac{P}{Q} = \frac{nmr_1}{mr_1}$ is large

(iv) M, the ratio $\frac{P}{R} = \frac{nmr_1}{nr_1}$ *is small*

The current I cannot be increased indefinitely because this will heat up the resistance coils and change their resistances. The sensitivity of the galvanometer itself is decreased if the resistance of the galvanometer G is too low.

Therefore, we have to increase the sensitivity by increasing n and decreasing m. In the ideal case, if $n = \infty$ and $m = 0$,

$$\left[\frac{I_g}{I}\right]_{max} = \frac{(r - r_1)}{(G + r_1)}$$

But this maximum sensitivity can never be realised in practices, because when $n = \infty$, the resistances in AB and AD are infinite. If m=0 the points A and C are short circuited.

If m = n = 1,

$$\frac{I_g}{I} = \frac{(r - r_1)}{2(G + r_1)}$$

This value is still half of the value in the ideal case. In practice, this is the condition for maximum sensitivity. However, for good sensitiveness of the bridge, n>1 and m<1. Thus the calendar rule is: The sensitivity of the bridge will be higher if the resistance in series with the unknown resistance is greater than the resistance connected in parallel to it.

The positions of the galvanometer and the battery can be interchanged but these two arrangements are not equally sensitive. Maxwell gave the following rule regarding the best arrangement of these.

Out of the battery and the galvanometer, the one having he higher resistance should be connected between the junction of the two highest resistances and the junction of the two lowest resistances.

2.4. Carey Foster Bridge

Description. The Carey Foster Bridge is a form of Wheat stone's bridge. It consists of a uniform wire AB of length 1 metre stretched on a wooden board(Fig 2.5)

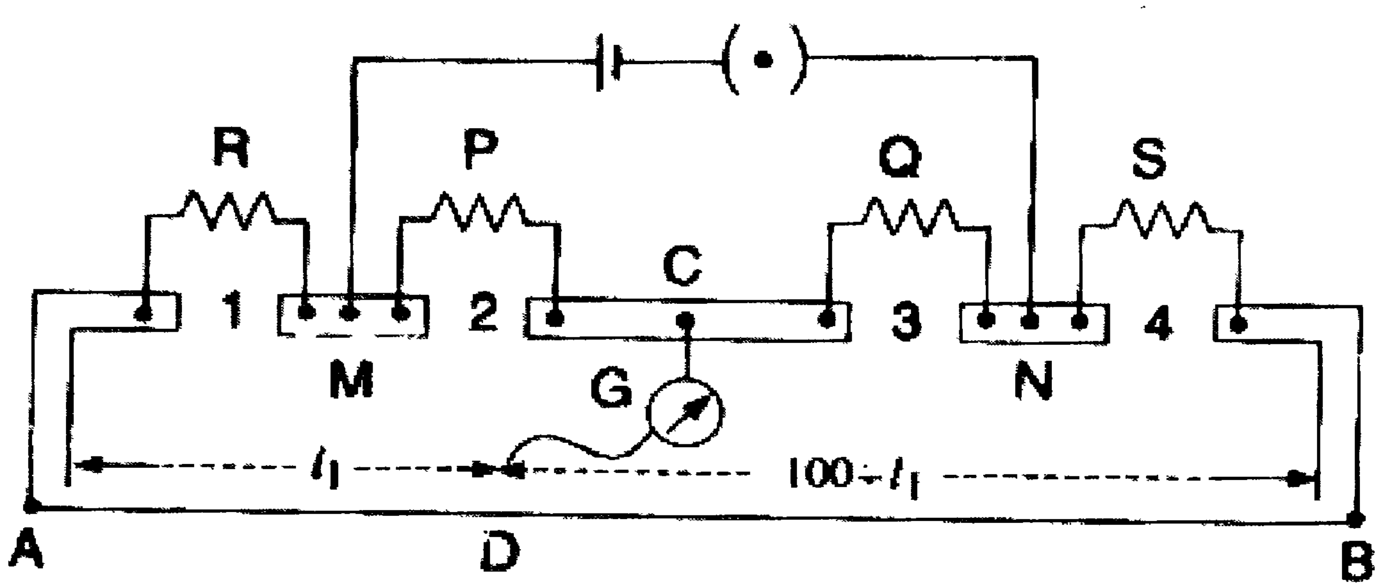


Fig 2.5

Two equal resistances P and Q are connected in gaps 2 and 3. The unknown resistance R is connected in gap 1. A standard resistance S, of the same order of resistance as R, is connected in gap 4. A Leclanche cell is connected across MN. A galvanometer G is connected between the terminal C and a sliding contact maker D.

Theory. The Contact maker is moved until the bridge is balanced. Let I_1 be the balancing length as measured from end A. Let α and β be the end resistances at A and B. Let ρ be the resistance per unit length of the wire.

From the principle of Wheat stone's bridge,

$$\frac{P}{Q} = \frac{R + \alpha + l_1 \rho}{S + \beta + \rho(100 - l_1)} \quad \ldots\ldots\ldots\ldots\ldots (1)$$

The resistance R and S are interchanged and the bridge is again balanced. The balancing length L_2 is determined from the same end A.
Then,

$$\frac{P}{Q} = \frac{R + \alpha + l_2 \rho}{S + \beta + \rho(100 - l_2)} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Fig 2.6 and 2.7 represent the equivalent Wheat stone's bridge circuit in the two cases.

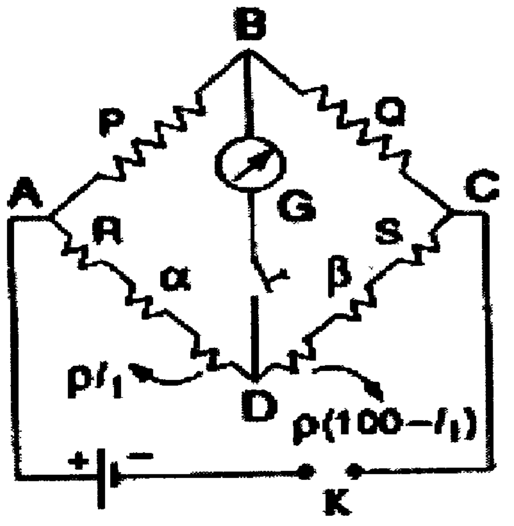


Fig 2.6

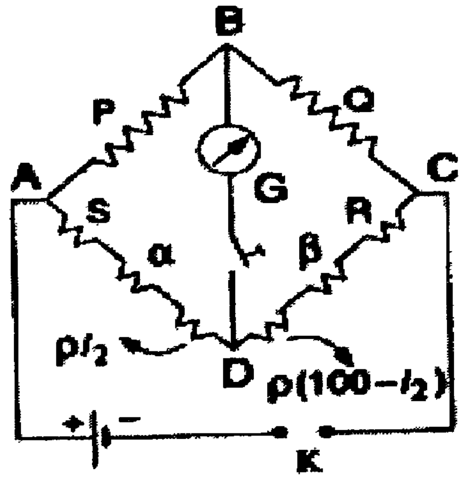


Fig 2.7

From Eqns. (1) and (2)

$$\frac{R+\alpha\ +l_{1\rho}\rho}{S+\beta+(100-l_1)\rho}=\frac{R+\alpha\ +l_{2\rho}\rho}{S+\beta+\rho(100-l_1)} \;\ldots\ldots(3)$$

Adding I to both sides of Eq.(3)

$$\frac{\mathrm{R}+\alpha\ +\mathrm{I}_{1\rho}+\mathrm{S}+\beta\ +100\rho-\ \mathrm{I}_{1\ \rho}}{\mathrm{S}+\beta+(100-l_1)\rho}=\frac{\mathrm{R}+\alpha\ +\mathrm{I}_{2\rho}+\mathrm{R}+\beta\ +100\rho-\ \mathrm{I}_2\ \rho}{\mathrm{R}+\beta+(100-l_1)\rho}$$

$$\frac{R+S+\alpha+\beta\ +100\rho}{S+\beta+(100-l_1)\rho}=\frac{R+S\ +\alpha+\beta\ +100\rho}{R+\beta+(100-l_1)\rho}$$

Since the numerators are equal, the denominators must be equal

$$S+\beta\ +100\rho-l_{1\rho}=R+\beta+100\rho-l_2\rho\ \ldots\ldots(4)$$

$$S-l_{1\rho}=R-\ l_2\,\rho$$

$$R=S+\rho(\,l_2-l_1)\ \ldots\ldots\ldots(5)$$

To find ρ. A standard resistance of 0.1Ω is connected in gap 1. A thick copper strip is connected in gap 4 i.e., R = 0.1Ω and S=0. The balancing length $l_1{}'$ is determined. The standard resistance and the thick copper strip are interchanged. The balancing length $l_2{}'$ is determined.

From Eq.(5) 0.1 = S+ ρ $(l_2{}'-l_1{}')$

(or) $\rho=\frac{0.1}{(l_2{}'-l_1{}')}$

Thus by knowing S andρ, the unknown resistance R is calculated.

Check your progress

1. In an experiment with Carey Foster bridge, the shift in the balance point is 5.4 cm when a thick copper strip and one ohm resistance are interchanged. The one ohm resistance is then replaced by an unknown resistance. Now the balance point shifts by 10 cm on interchanging calculate the unknown resistance.

Ans: ---

2.5 Potentiometer

Principle: A potentiometer is device for measuring or comparing potential differences. A potentiometer can be used to measure any electrical quantity which can be converted in to a proportionate D.C potential difference. It consists of a uniform wire AB of length usually 10 metres stretched on a wooden board by the side of a metre scale. The wires of length one mere each are joined in series and connected between the points A and B. The wires used have a low temperature co-efficient of resistance. A steady current is passed through the wire AB with the help of a constant source of E.M.F (Fig 2.8)

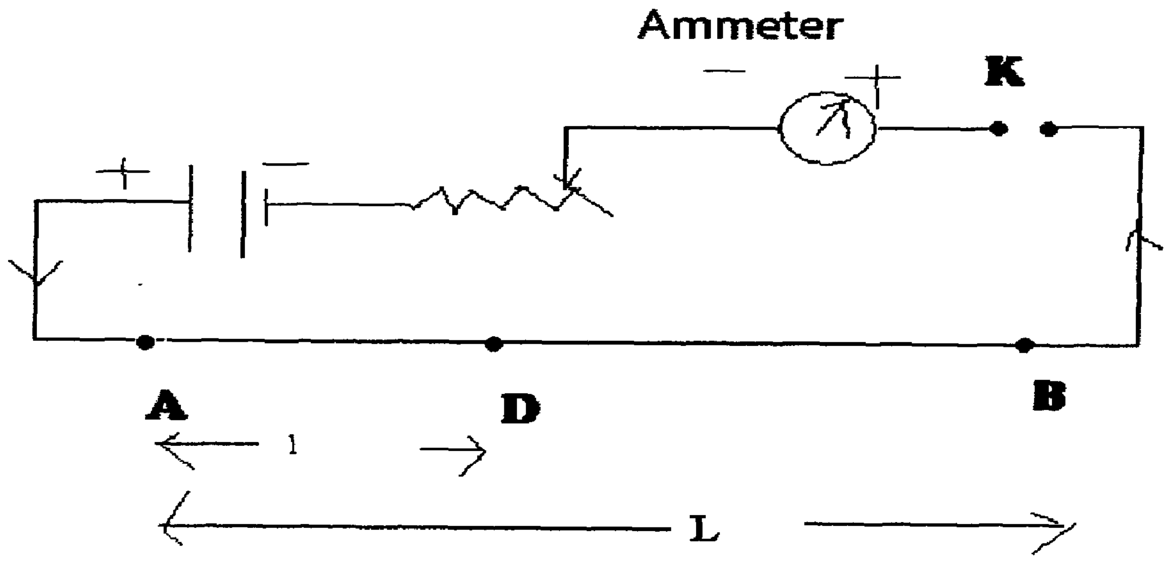


Fig 2.8

Let the resistance per unit length of the potentiometer wire be ρ and the steady current passing through the wire be I amperes.

AB = L cm

AD = l cm

PD across AB = L ρI

PD across AD = l ρI

$$\frac{PD\ across\ AB = L\rho I}{PD\ across\ AD = l\rho I} = \frac{L}{l}$$

PD across AD = $\frac{l}{L}$ x $PD\ across\ AB$

Thus, for a steady current passing through the potentiometer wire AB, the potential difference across any length is proportional to the length of the wire.

2.6. Determination of the Internal Resistance of a Cell

The given cell whose internal resistance is to be determined is connected is the circuit as shown in Fig.2.9. A resistance box R is connected parallel to the cell and AB is the potentiometer wire. A steady current is passed through the wire with the help of a battery.

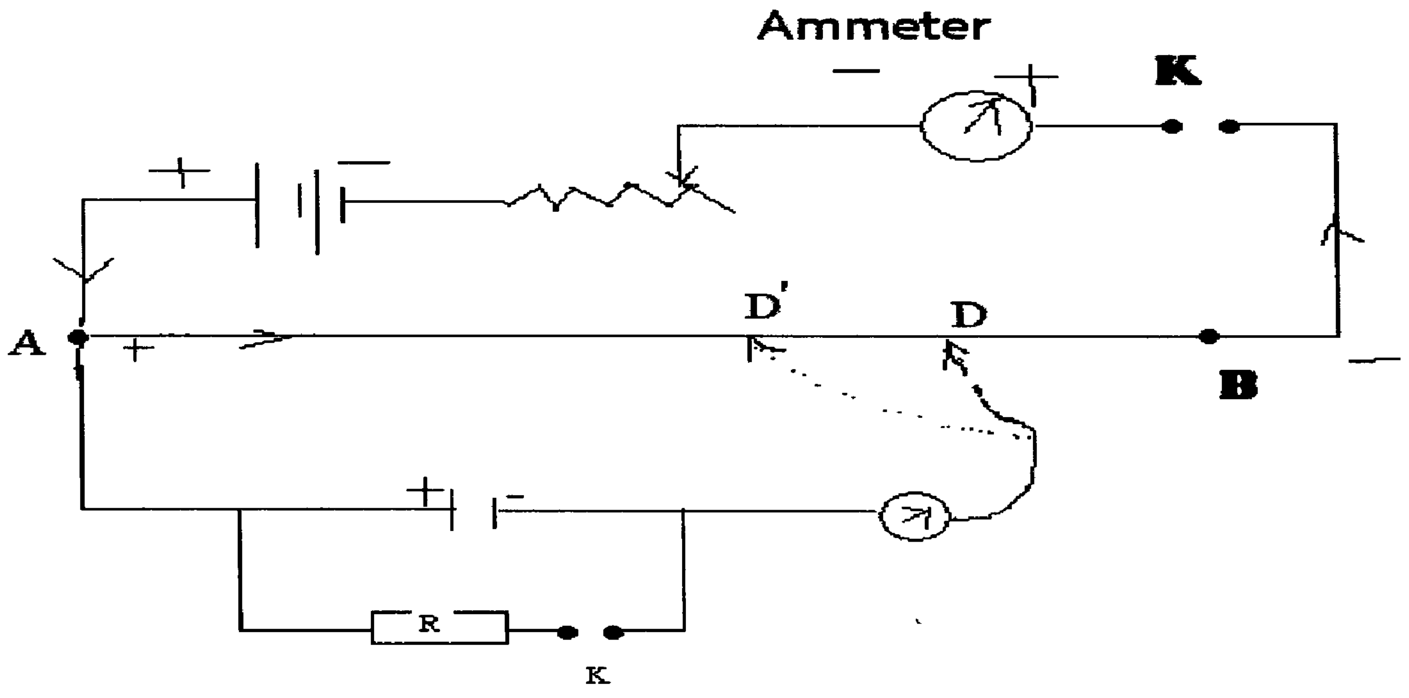


Fig 2.9

If E is the EMF of the cell, r the internal resistance and V the potential difference across the cell when supplying a current I through the external resistance R

Then $$V = IR$$

and $$E = I(R + r)$$

$$\frac{E}{V} = \frac{R+r}{R} = 1 + \frac{r}{R}$$

$$r = \left(\frac{E-V}{V}\right) R \qquad ......(1)$$

Initially with the key K_1 open, the balancing length l_1 (=AD) is determined. Keeping the current through the potentiometer wire the same, the key K1 is closed, and with a resistance R from the resistance box, the balancing length l_2 (=AD') is determined. EMF, E α PD across l_2

$$\frac{E}{V} = \frac{l_1}{l_2}$$

or $$\frac{E-V}{V} = \frac{l_1 - l_2}{l_2}$$

Substituting this value in equation (i)

$$r = \left(\frac{l_1 - l_2}{l_2}\right) R$$

The value of r is determined with different values of R and the mean value of the internal resistance is calculated.

2.7 Measurement of potential and Calibration of voltmeter (low Range)

A battery is connected to the ends of a potentiometer wire through a rheostat and a plug key. To the positive end A, the positive terminal of a Daniel cell of steady emf. 1.08 V is connected. The negative terminal of the Daniel cell is connected to the jockey through a galvanometer and a high resistance [fig 2.10)] . The jockey is pressed at the 540th centimetre (c) from the positive end A. There will be deflection in the galvanometer which is reduced to zero by adjusting the rheostat in the primary circuit. Now the potentiometer wire has a potential difference of 1.08 V for 540 cm of its length. This corresponds to p.d. of $\frac{1.08}{540} = 0.042\ V\ per\ cm.$ of the potentiometer wire. In this condition, the potentiometer of length 1000cm can balance an emf or p.d of 2 V. If an unknown p.d is to be measured, it is applied between the end A and the jockey through a galvanometer in such a way what the positive of the unknown p.d. to be measured, is applied between the end A and the jockey through a galvanometer in such a way that the positive of the unknown p.d. is connected to the positive end A of the potentiometer wire. By pressing the jockey at various points, the balancing point

and hence the balancing length *l* cm of the potentiometer wire are determined. The p.d across *l* balances the unknown p.d. of e volt. Then, e = 0.002**l* volt.

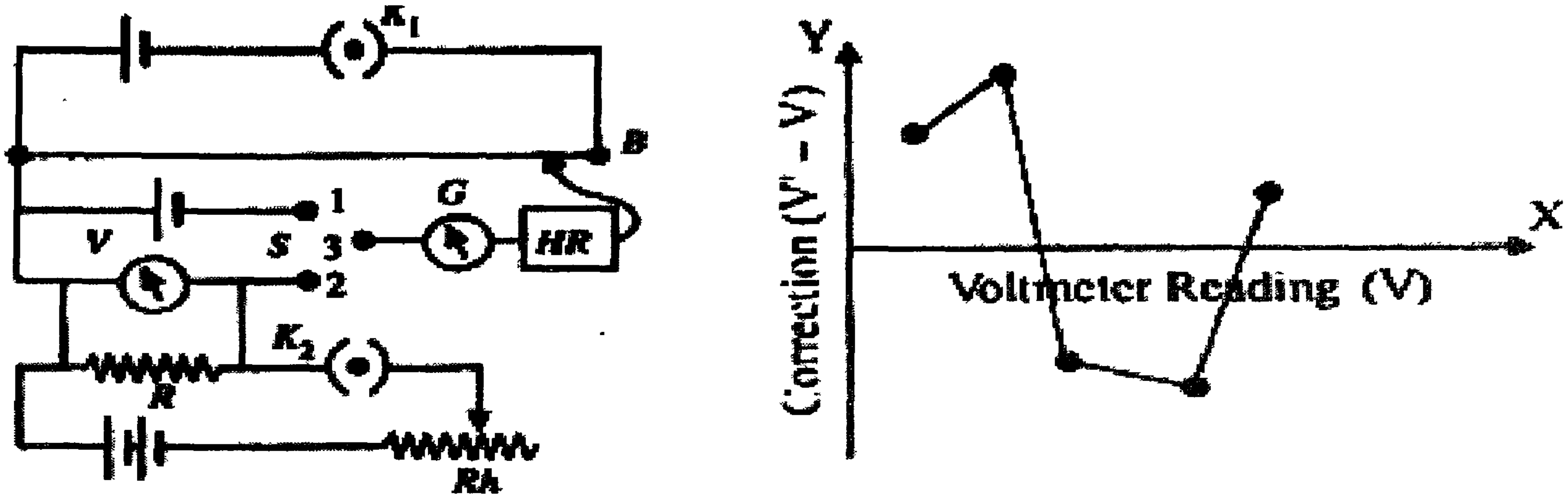


Fig 2.10

[Balancing point is that point on which, if the jockey is pressed the galvanometer shows zero deflection. Then distance of this point from the end A is known as the balancing length.]

The potentiometer adjusted to have a p.d. of 0.002 volt per cm as above, may be used to calibrate a voltmeter in the range o-2V. For this it is connected as the secondary circuit between the positive end A and the jockey. Contacts are made with the jockey at various points between A and B so that the voltmeter shows different readings. In each case, the length 1 cm, of the wire between the end A and the point of contact J is measured. When the voltmeter shows a reading V, the actual p.d. across it is V' = 0.002l, hence the correction to be applied to the voltmeter is calculated as V' –V.

A graph may be plotted with various voltmeter reading on the X axis and the corresponding corrections on the Y axis. The points plotted are joined by straight lines. The graph is called the calibration graph. The correction to be applied for any voltmeter may be found from this graph.

If a larger p.d. of 5 V or 10 V is to be measured or if a voltmeter is to be calibrated for higher ranges, the potentiometer may be arranged to have α p.d. of 0.005 or 0.01 volt per cm. For this battery connected in the primary circuit must have an emf greater than 5V or 10V and the length of the potentiometer wire balancing the emf of the Daniel cell must be 216 cm, or 107 cm.

2.7.1 Calibration of Voltmeter (High Range)

Using a Daniel cell, the potentiometer is adjusted to have a potential difference of 0.002 Volt/cm as for the calibration of low range voltmeter. To calibrate a high range voltmeter (say 0 – 30 V), a variable D.C. power supply

capable of providing a voltage V_c exceeding 30 V is taken, and the given high range voltmeter is connected across it. Two resistance boxes P and Q are also connected across the power supply. The positive terminal of P is connected to the positive terminal of the potentiometer, and the other terminal of P is connected to the jockey of the potentiometer through a galvanometer G and high resistance HR (Fig 2.11)

The D.C. power supply is adjusted for a low voltmeter reading V_m. With P + Q = 10,000Ω and a low resistance in P, a p.d. less than 2 V is obtained across P, and balanced on the potentiometer wire. Let the balance length be L cm. Thus p.d. across P = P.D across L cm of potentiometer wire.

$$\text{Therefore } \frac{V_C P}{(P+Q)} = 0.022 * L$$

When V_c is the correct voltage for the voltmeter reading V_m.

$$\text{Therefore } V_C = \frac{0.002\, L\, (P+Q)}{P}$$

The correction for the voltmeter reading is ($V_c - V_m$). The experiment is repeated for various reading V_m of the voltmeter. A calibration graph is drawn with voltmeter reading V_m in the X axis, and the corresponding corrections ($V_c - V_m$) in the Y axis.

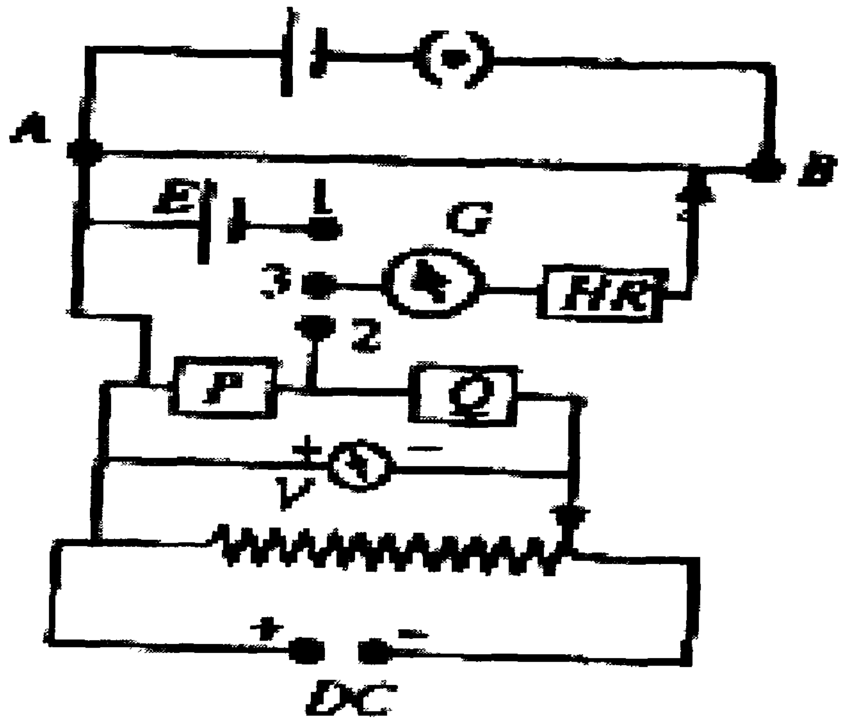


Fig 2.11

2.7.2 Measurement of Current and Calibration of ammeter

An accumulator is connected to the end of the potentiometer wire through a lug key. To the positive end A, the positive terminal of a Daniel cell of steady emf 1.08 V is connected. The negative terminal of a Daniel cell is connected to jockey through a galvanometer and a high resistance [Fig.2.12]. The balancing point and the balancing length l_0 of the potentiometer wire (balancing the emf. 1.08 V of the Daniel cell) are found. Now there is a.p.d. of $\frac{1.08}{l_0}$ volt per cm of the potentiometer wire. After this the Daniel cell is disconnected.

A separate secondary circuit is connected with the given ammeter in series with a battery, plug key rheostat and a standard resistance r (1 ohm or 2 ohm). The p.d across the standard resistance is applied in opposition between the positive end A of the potentiometer wire and the jockey through a galvanometer

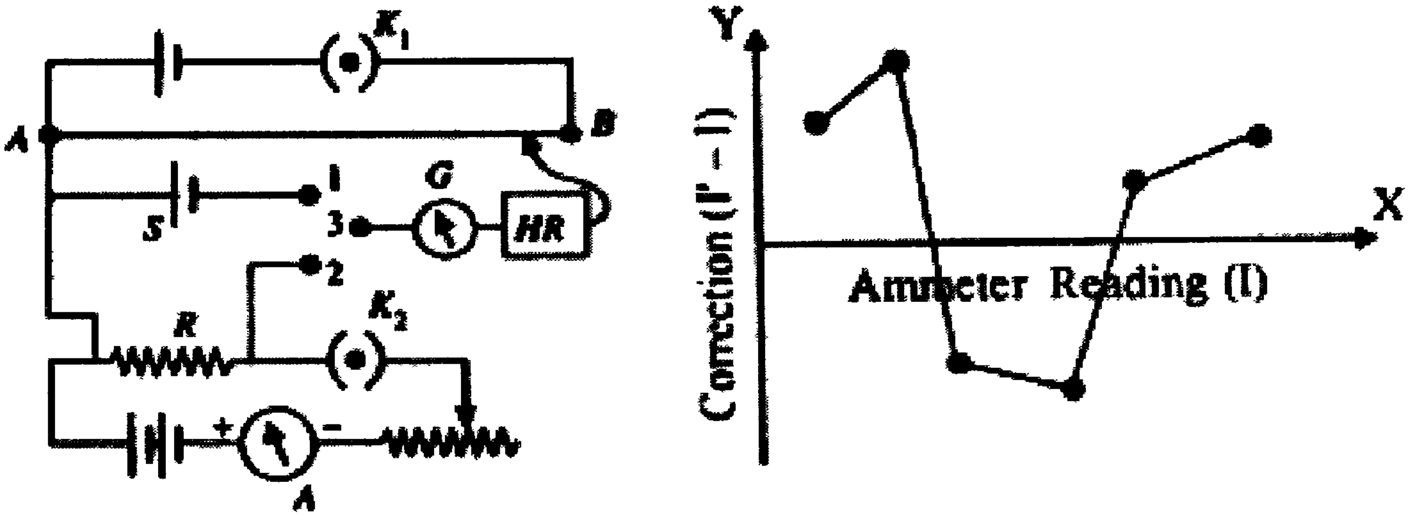


Fig 2.12

and a high resistance . The rheostat is adjusted so that the ammeter reads various currents. For each reading I of the ammeter, the corresponding balancing length l is determined. Now the p.d. across the standard resistance r is balanced against the p.d. across the length l

Therefore P.d. across $r = \frac{1.08}{l_0} \text{ x } l$

The current through $r = \frac{1.08}{l_0} * \frac{L}{r} = l'$ which is the correct current through the ammeter. The correction to be applied to the ammeter reading I is calculated as I' – I.

A calibration graph is drawn with various ammeter reading on the X-axis and the corresponding corrections on the Y –axis. The correction to be applied for any ammeter reading may be found from this graph.

2.8 See beck Effect

When two dissimilar metal wires are joined together so as to form a closed circuit and if the two junctions are maintained at different temperatures, an emf is developed in the circuit (Fig.2.13). This causes a current to flow in the circuit as indicated by the deflection in the galvanometer G. This phenomenon is called the See beck effect. This arrangement is called a thermocouple. The emf developed is called thermo emf. The thermo emf so developed depends on the temperature difference between the two junctions and the metals chosen for the couple. See beck arranged the metals in a series as follows:

Bi, Ni, Pd, Pt, Cu, Mn, Hg, Pb, Sn, Au, Ag, Zn, Cd, Fe, Sb.

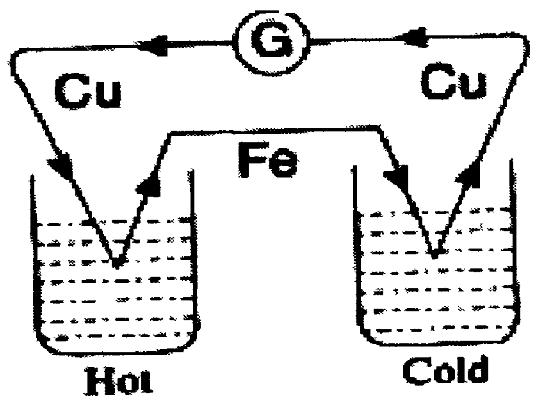


Fig 2. 13

When a thermocouple is formed between any two of them, the thermoelectric current flows through the hot junction from the metal occurring earlier to the metal occurring later in the list. The more removed are the two metals in the list, the greater is the thermo emf developed. The metals to the left of Pb are called thermoelectrically negative and those to its right are thermoelectrically positive.

Check your progress
2. What is a thermo couple? Mention few thermoelectrically positive and negative metals. Ans: --- --- --- --------------------------------

2.8.1 Laws of thermo e.m.f:

(i) Law of Intermediate Metals. The introduction of any additional metal into an thermoelectric circuit does not alter the thermo emf provided the metal

introduced is entirely at the same temperatures the point at which the metal is introduced.

If ${}_aE_b$ is the emf for a couple made of metals A and B, and ${}_bE_c$ that for the couple of metals B and C, then the emf for couple of metals A and C is given by.

$$ {}_aE_c = {}_aE_b + {}_bE_c $$

(ii) **Law of Intermediate Temperatures.** The thermo emf $E_1{}^3$ of a thermocouple whose junctions are maintained at temperatures T_1 and T_3 is equal to the sum of the $E_1{}^2$ *and* $E_2{}^3$ when the junctions are maintained at temperatures T_1, T_2 and T_2, T_3 respectively. Thus

$$E_1{}^3 = E_1{}^2 + E_2{}^3$$

2.9. Measurement of Thermo EMF using Potentiometer

Thermo emfs are very small, of the order of only a few millivolts. Such small emfs are measured using a potentiometer. A ten-wire potentiometer of resistance R is connected in series with an accumulator and resistance boxes P and Q (Fig 2.14). A standard cell of emf E is connected in the secondary circuit. The positive terminal of the cell is connected to the positive of Q. The negative terminal of the cell is connected to a galvanometer and through a key to the negative of Q.

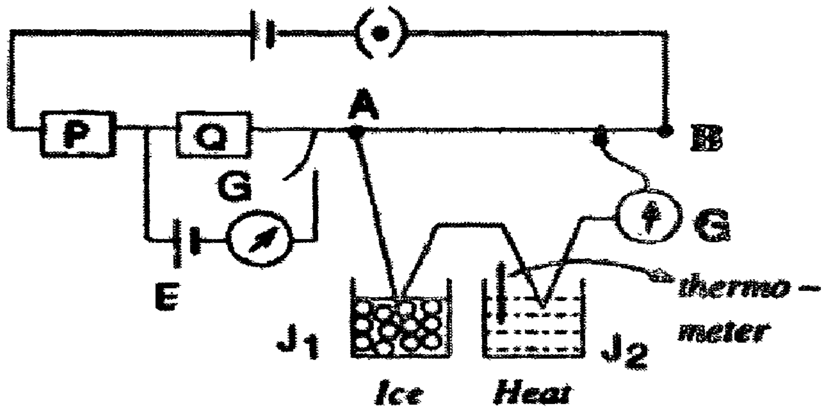


Fig 2.14

A resistance of 100 ER ohms is taken in Q. The resistance in P is adjusted so that on closing the key, there is no deflection in the galvanometer. Now, the PD across 100 ER ohms is equal to E.

PD across R ohms of the Potentiometer $\Big\}$ $\frac{ER}{100\,ER}$ *volt* $= \frac{1}{100}$ *volt* $= 10$ *millivolt.*

Thus the fall of potential per metre of the potentiometer wire is 1 millivolt. So we can measure thermo emf up to 10 millivolt.

Without altering the resistances in P and Q, the positive of the thermocouple is connected to the positive e terminal of the potentiometer and the negative of the thermocouple to a galvanometer and jockey. One junction is kept in melting ice and the other junction in an oil bath or in a sand bath. The jockey is moved till a balance is obtained against the small emf e of the thermocouple. Let AJ = l cm is the balancing length. Then,

Thermo emf e= $\frac{1}{100} l$ *millivolt.*

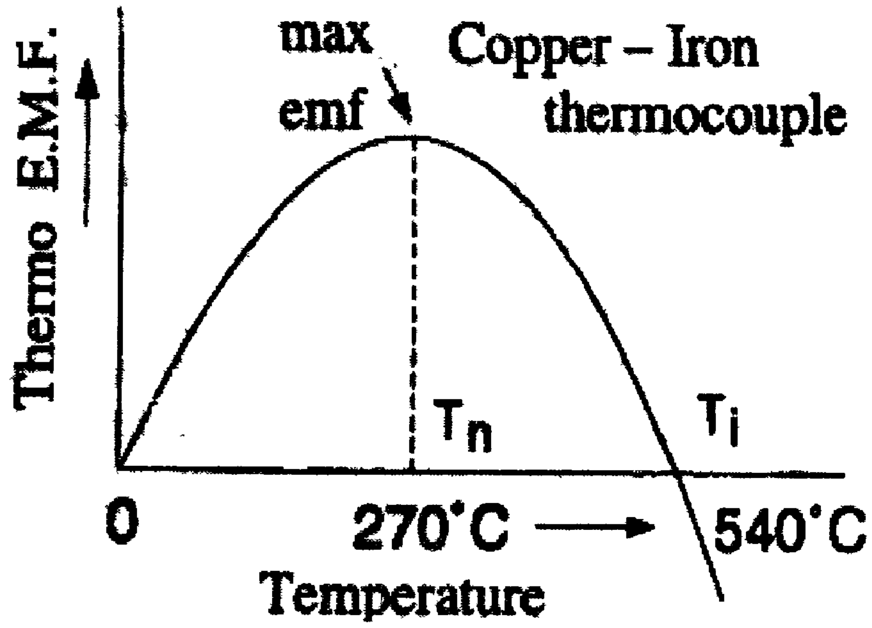


Fig 2.15

Keeping the cold junction at 0° C, the hot junction is heated to different temperatures. The thermo emf generated is determined for different temperatures of the hot junction. A graph is drawn between thermo emf and the temperature of the hot junction (Fig.2.15). The graph is a parabolic curve.

The thermo emf E varies with temperature according to E = $at+bt^2$, where a and b are constants. The thermo emf increases as the temperature of the hot junction increases, reaches a maximum value at a certain temperature T_n, then decreases to zero at a particular temperature T_i. On further increasing the difference of temperature, emf is reversed in direction.

For a given temperature of the cold junctions, the temperature of the hot function for which the thermo emf becomes maximum is called the neutral temperature (T_n) *for the given thermocouple.*

For a given temperature of the cold junction, the temperature of the hot junction for which the thermo emf becomes zero and changes its direction is called the inversion temperature (T_i) *for the given thermocouple.*

T_n is a constant for the pair of metals. T_i is variable. T is as much above the neutral temperature as the cold junction is below it.

2.10. Peltier Effect

Consider a copper – iron thermocouple (Fig.2.16). When a current is allowed to pass through the thermocouple in the direction of arrows (from A to B), heat is absorbed at the junction B and generated at the junction A. This absorption or evolution of hat at a junction when a current is sent through a thermocouple is called Peltier effect. The Peltier effect is a reversible phenomenon, if the direction of the current is reversed, then there will be cooling at the junction A and heating at the junction B.

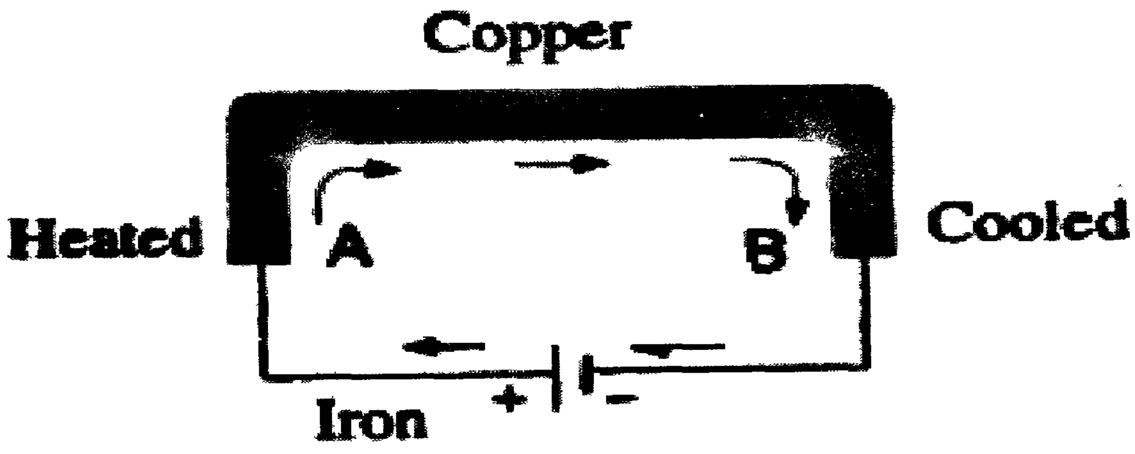


Fig 2.16

When an electric current is passed through a closed circuit made up of two different metals, one junction is heated and the other junction is cooled. This is known as Peltier effect.

The amount of heat H absorbed or evolved at a junction is proportional to the charge q passing through the junction i.e.,

$$H \propto q \text{ or } H \propto It$$

Or

$$H = \pi I t$$

Where π is a constant called Peltier Coefficient

When I = 1 A and t = 1 sec, then H = π

The energy that is liberated or absorbed at a junction between two dissimilar metals due to the passage of unit quantity of electricity is called Peltier coefficient.

It is expressed in joule/coulomb i.e., volt. The Peltier coefficient is not constant but depends on the temperature of the junctions.

2.10 Thomson Effect

Consider a copper bar AB heated in the middle at the point C (Fig.2.17). A current is passed from A to B. It is observed that heat is absorbed

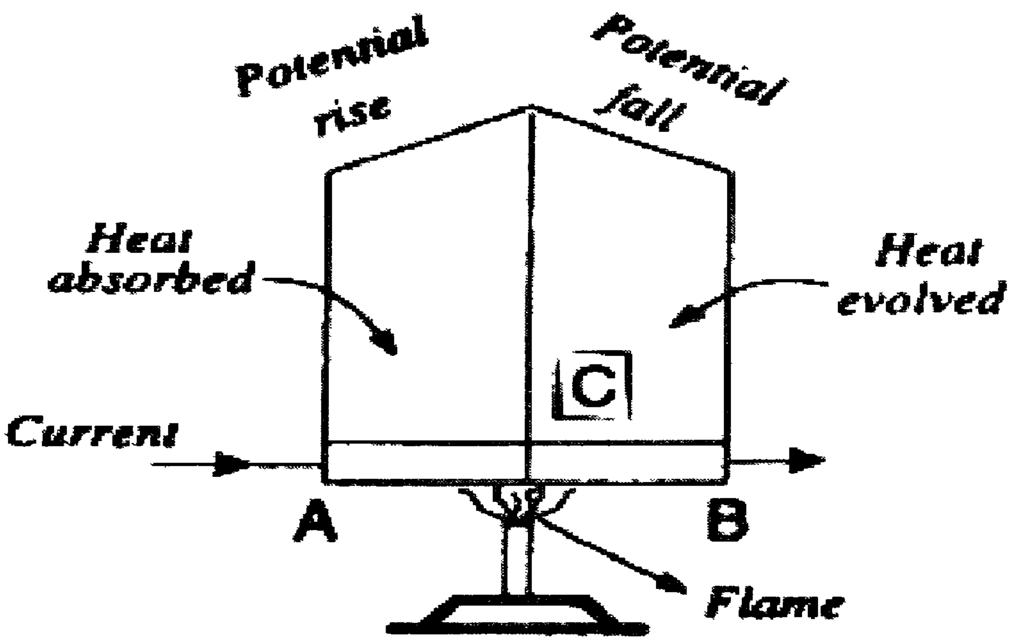


Fig 2.17

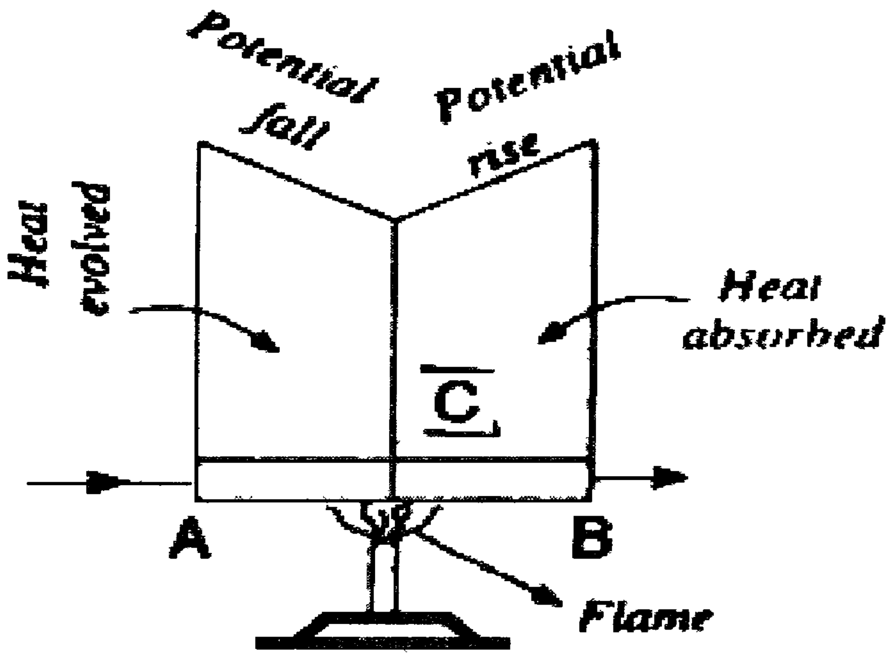


Fig 2.18

in the part AC and evolved in the part CB. This is known as Posit vive Thomson effect. Similar effect is observed in metals like Ag, Zn, Sb, and Dd.

In the case of an iron bar AB, heat is evolved in the part AC and absorbed in the part CB (Fig 2.18). This is known as Negative Thomson effect. Similar effect is observed in metals like Pt, Ni, Co and Bi.

For lead, the Thomson effect is zero.

The Thomson effect is reversible.

In the case of copper, the hotter parts are at a higher potential than the colder ones. It is opposite in the case of iron. Heat is either absorbed or evolved when current passes between two points having a difference of potential. Therefore, the

passage of electric current through a metal having temperature gradient results in an absorption or evolution of heat in the body of the metal.

When a current flows through an unequally heated metal, there is an absorption or evolution of heat throughout in the body of the metal. This is known as 'Thomson effect'.

Thomson Coefficient: The Thomson coefficient σ of a metal is defined as the amount of heat energy absorbed or evolved when a charge of 1 coulomb flows in the metal between two points which differ in temperature by 1° C.

Thus, if a charge of q coulomb flows in a metal between two points having a temperature difference of 1° C, then

Heat energy absorbed or evolved = σq joule.

But if E volt be the Thomson emf developed between these points then this energy must be equal to Eq joule.

$$\sigma q = Eq$$

$$\sigma = E.$$

Thus the Thomson coefficient of a metal, expressed in joule per coulomb per ° C, is numerically equal to the emf in volt, developed between two points differing in temperature by 1°C.

Hence is may also be expressed in volt per ° C.

σ is not a constant for a given meal. It is a function of temperature.

Check your progress
3. Define the terms: 'Neutral Temperature' and 'Temperature of inversion' Ans: --- --- --- ---

2.11.1 Thermodynamics of Thermocouple
(Expressions for Peltier and Thomson coefficients)

Consider a thermocouple consisting of two metals A and B, Let T and T +dT be the temperatures of the cold and ho junctions respectively (Fig 2.19). Let π and π +dπ be the Peltier coefficients for the pair at the cold and hot junctions. Let σ_a *and* σ_b be the Thomson coefficients for the metals A and B respectively, both taken as positive. When a charge flows through the thermocouple, heat will be absorbed and evolved at the junctions due to Peltier effect and all along the meta due to Thomson effect.

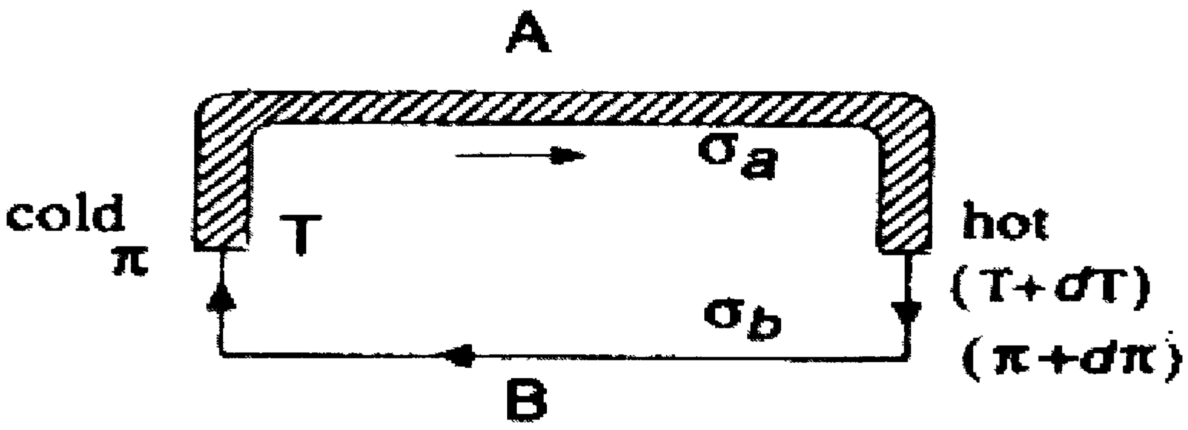


Fig 2.19

Let 1 coulomb of charge flow through the thermocouple in the direction from A to B at the hot junction.

Heat energy absorbed due to Peltier effect at the hot junction = (π+dπ) joules.

Heat energy evolved due to Peltier effect at the cold junction = π joules

Heat energy absorbed in the metal A due to Thomson effect $=\sigma_a dT\ joules$

Heat energy evolved in the metal B due to Thomson effect $=\sigma_b\ dT\ joules$

Net heat energy absorbed in the thermocouple

$$=(\pi+d\pi-\pi) + \sigma_a dT - \sigma_b dT$$

$$= d\pi + (\sigma_a - \sigma_b)\ dT$$

The energy is used in establishing a P.D dE i n the thermocouple

$$dE= d\pi + (\sigma_a - \sigma_b)\ dT \quad ....(1)$$

since the Peltier and Thomson effects are reversible, the thermocouple acts as a reversible heat engine. Here.

(i) The heat energy (π +d π) joule is absorbed from the source at (T+dT) K and $\sigma_a dT$ joule is absorbed in metal A at mean temperature TK.

(ii) Also π joule is rejected to sink at T K and σ_bdT joule is given out in metal B at the mean temperature T K.

Applying Carnot's theorem, we have

$$\frac{\pi + d\pi}{T + dT} + \frac{\sigma_a dT}{T} = \frac{\pi}{T} + \frac{\sigma_b dT}{T}$$

$$\frac{\pi + d\pi}{T + dT} - \frac{\pi}{T} = \frac{(\sigma_b - \sigma_a)dT}{T}$$

$$\frac{\pi T + d\pi T - \pi T - \pi dT}{T(T + dT)} = \frac{(\sigma_b - \sigma_a)dT}{T}$$

$$d\pi.T - \pi.dT = (\sigma_b - \sigma_a)dT(T + dT)$$

$$d\pi.T - \pi.dT = (\sigma_b - \sigma_a)TdT + (\sigma_b - \sigma_a)dT^2$$

$$(d\pi.T - \pi.dT) = (\sigma_b - \sigma_a)T.dT$$

[*Neglecting* $((\sigma_b - \sigma_a)dT^2]$

$$T(d\pi + (\sigma_a - \sigma_b)dT] = \pi dT$$

But $\quad d\pi + (\sigma_a - \sigma_b)dT = dE \quad$ from Eq.(1)

$$TdE = \pi . dT$$

$$\pi = T.\frac{dE}{dT} \quad ....(2)$$

The quantity (dE/dT) is called the thermoelectric power (P)
Thermoelectric power (P) is defined as the thermo emf per unit difference of temperature between the junctions.
Peltier coefficient = (Absolute temperature) x (thermoelectric power)

Differentiating Eq.(2), $\frac{d\pi}{dT} = T\frac{d^2E}{dT^2} + \frac{dE}{dT}$

Substituting the value of (dE/dT) from Eq.(1).

$$\frac{d\pi}{dT} = T\frac{d^2E}{dT^2} + \frac{dE}{dT} + (\sigma_a - \sigma_b)$$

$$(\sigma_a - \sigma_b) = -T.\frac{d^2E}{dT^2}$$

$$(\sigma_b - \sigma_a) = T.\frac{d^2E}{dT^2} ....(3)$$

If the first metal in the thermocouple is lead, then $\sigma_a = 0$

$$\sigma_b = T.\frac{d^2E}{dT^2} \quad ....(4)$$

Thomson coefficient = (absolute temperature of the cold junction) x (first derivative of thermoelectric power).

From Eq.(3) $\frac{d^2E}{dT^2} = \frac{(\sigma_b - \sigma_a)}{T}$ or $\frac{d}{dT}\left(\frac{dE}{dT}\right) = \frac{(\sigma_b - \sigma_a)}{T}$

Putting dE/dT from Eq. (2),we have

$$\frac{d}{dT}\left(\frac{\pi}{T}\right) = \left(\frac{\sigma_b - \sigma_a}{T}\right) = 0 \quad ....(5)$$

This gives the relation between Peltier and Thomson's coefficient.

2.12 Thermo electric Diagrams

A thermocouple is formed from two metals A and B. The difference of temperature of the junction is TK. The thermo emf E is given by the equation.

$$E = aT + bT^2$$

A graph between E and T is a parabola.

$$\frac{dE}{dT} = a + 2bT$$

dE/dT is called thermoelectric power.

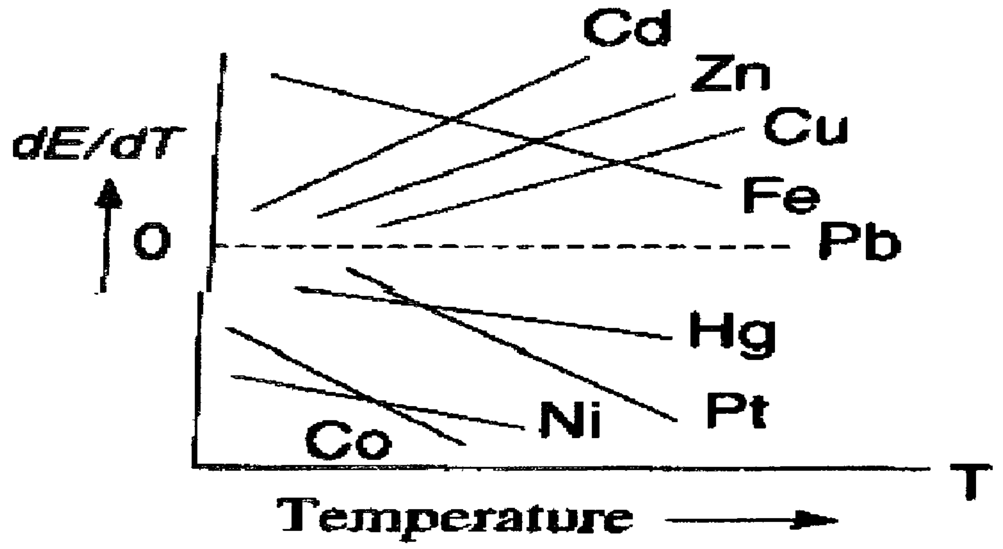


Fig 2.20

A graph between thermoelectric power (dE/dT) and difference of temperature T is a straight line. This graph is called the thermo-electric power line or the thermo – electric diagram. Thomson coefficient of lead is zero. So generally thermo electric lines are drawn with lead as one metal of the thermocouple. The thermoelectric line of a Cu=Pb couple has a positive slope while that of Fe-Pb couple has a negative slope. Fig. 2.20 shows the power lines for a number of metals.

2.13 Let us sum-up

- **Kirchhoff's Law 1-** In any network of conductors, the algebraic sum of the currents meeting at any point is zero i.e., $\Sigma I = 0$
- **Kirchhoff's Law 2-**The algebraic sum o the products of the current and resistance in any closed loop of a circuit is equal to the algebraic sum of electromotive forces (e.m.fs) acting in that loop. i.e., Σ IR= ΣE.
- The sensitivity of the bridge will be higher if the resistance in series with the unknown resistance is greater than the resistance connected in parallel to it.
- **Potentiometer** is a device used to measure potential difference.

- **Seeback effect**- When two dissimilar metal wires are joined together so as to form a closed circuit and if the two junctions are maintained at different temperatures, an emf is developed
- **Law of Intermediate Metals.** The introduction of any additional metal into an thermoelectric circuit does not alter the thermo emf provided the metal introduced is entirely at the same temperatures the point at which the metal is introduced.
- **Law of Intermediate Temperatures.** The thermo emf $E_1{}^3$ of a thermocouple whose junctions are maintained at temperatures T_1 and T_3 is equal to the sum of the $E_1{}^2$ *and* $E_2{}^3$ when the junctions are maintained at temperatures T_1, T_2 and T_2,T_3 respectively. Thus

$$E_1{}^3 = E_1{}^2 + E_2{}^3$$

- **Neutral Temperature**: *For a given temperature of the cold junctions, the temperature of the hot function for which the thermo emf becomes maximum is called the neutral temperature* **(T_n)** *for the given thermocouple.*
- **Inversion Temperature:** *For a given temperature of the cold junction, the temperature of the hot junction for which the thermo emf becomes zero and changes its direction is called the inversion temperature* **(T_i)** *for the given thermocouple.*

- **Peltier effect** - The energy that is liberated or absorbed at a junction between two dissimilar metals due to the passage of unit quantity of electricity is called Peltier coefficient. It is expressed in joule/coulomb i.e., volt.
- **Thomson effect** - When a current flows through an unequally heated metal, there is an absorption or evolution of heat throughout in the body of the metal.

2.14 Unit –end exercises

1. State and explain Kirchhoff's Laws
2. Describe the measurement of current using potentiometer.
3. State Kirchhoff's laws of distribution of currents in an electrical network. Apply these laws to deduce the condition of balance of a Wheat stone's bridge.
4. Using Kirchhoff's laws, derive an expression for the sensitivity of Whetstone's bridge. How do the positions of cell and galvanometer affect the sensitivity?

5. Explain the theory of a potentiometer. How will you use it to find the internal resistance of a cell?
6. Two batteries of 7 volts and 13 volts and internal resistance 1ohm and 2 ohms respectively are connected in parallel with a resistance of 12 ohms. Find the current through each branch of the circuit and the potential difference across 12 ohms resistance.
7. Describe Carey-Foster bridge experiment with necessary theory to determine the resistance of a conductor.
8. How will you calibrate an Ammeter using Potentiometer?
9. Explain how a low range voltmeter could be calibrated using a potentiometer.
10. State the laws of (i) intermediate metals and (ii) intermediate temperatures
11. Define the terms : 'Neutral Temperature and Temperature of inversion'
12. What is thermoelectric power diagram
13. Explain the terms (i) Thermoelectric power (ii) Peltier co-efficient and (iii) Thomson co-efficient

2.15 Problems for discussion

1. A battery of e.m.f., 6 volts and internal resistance 5 ohm is joined in parallel with another of e.m.f., 10 volts and internal resistance 1 ohm and the combination sends a current through each battery.

2. Twelve conductors each of resistance 3 ohm are connected to form a skeleton cube. A battery of e.m.f. 6 V and internal resistance 0.5 ohm is connected between two diagonally opposite corners of the cube. Find the equivalent resistance of the cube and the current supplied by the battery.

3. A Thermo-couple is made of iron and constantan. Find the emf developed per °C difference of temperatures between the junctions, given that thermo-emfs of iron and constantan against platinum are + 1600 and -3400 μV pr 100 °C difference of temperature.
4. In copper, there are 10^{22} free electrons per cm^3 all of which contribute to current of 1 ampere in a wire of copper of 0.01 cm^2 cross-sectional area.
   (i) What is the average speed of electrons in the copper wire
   (ii) What is the electric field in the wire? Given specific resistance of copper = 1.6×10^{-8} ohm-metre.
5. Twelve conductors each of resistance 2 ohm are connected to form a cube. A battery of e.m.f. 10V is connected between the two diagonally opposite corners of the cube. Calculate the equivalent resistance of the framework and the current through each resistance.

2.16 Answers for check your progress and Problems for discussion

Check your progress:

Answer 1:

$\rho = 1/5.4$ ohm / cm

$R = S + \rho(l_2 - l_1) = 1.85$ ohm.

Answer 2:

When two dissimilar metal wires are joined together so as to form a closed circuit and if the two junctions are maintained at different temperatures, an emf is developed in the circuit. This arrangement is called a thermocouple

Bi, Ni, Pd, Pt, Cu, Mn, Hg thermoelectrically positive metals

Sn, Au, Ag, Zn, Cd, Fe, Sb. thermoelectrically negative metals

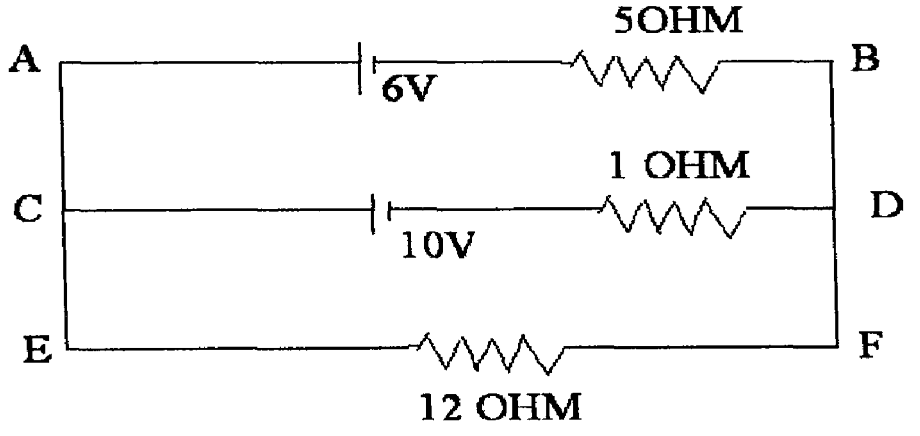
Answer 3:

Neutral Temperature: *For a given temperature of the cold junctions, the temperature of the hot function for which the thermo emf becomes maximum is called the neutral temperature* (T_n) *for the given thermocouple.*

Inversion Temperature: *For a given temperature of the cold junction, the temperature of the hot junction for which the thermo emf becomes zero and changes its direction is called the inversion temperature* (T_i) *for the given thermocouple*

Problems for discussion:

Solution1:



Applying Kirchhoff' law second law to the mesh A B F E A

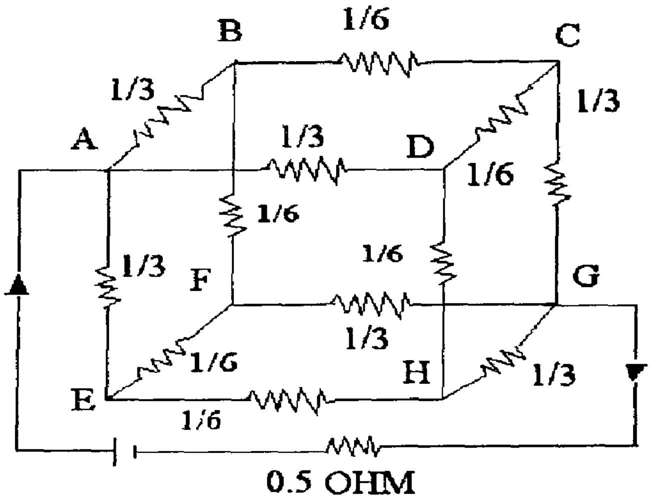
$$I_1 \times 5 + (I_1+I_2) \times 12 = 6 \qquad ......(1)$$

Applying Kirchhoff' law second law to the mesh C E F D C

$$I_2 \times 1 + (I_1+I_2) \times 12 = 10 \qquad .....(2)$$

Solving 1 & 2 $I_1 = -6/11$ ampere & $I_2 = 14/11$ ampere

Solution 2:



Let *I* be the current supplied by the battery. The current *I* enter the system at *A* and leaves it at *G*. The distribution of current in the various branches is as shown in the figure.

Applying KVL to the closed loop ABFGA

$$I/3 \times 3 + I/6 \times 3 + I/3 \times 3 + 0.5\, I = 6$$

$$I = 2A$$

Let X be the equivalent resistance of the frame work. Then,

$$(X + 0.5) \times I = 6; \quad X = 2.5\ \Omega.$$

Solution 3:

$$E_{Con}^{Fe} = E_{Pt}^{Fe} + E_{Con}^{Pt} = E_{Pt}^{Fe} - E_{Pt}^{Con}$$

$$E_{Pt}^{Fe} = 16\mu \frac{V}{^{o}C}$$

$$E_{Pt}^{Con} = -\frac{34\mu V}{^{o}C}$$

$$E_{Con}^{Fe} = \frac{50\mu V}{°C}$$

4. (i)6.25×10^{-4} ms^{-1}

   (ii) $J = \sigma E = \frac{1}{\rho} E$

   $E = \rho J = 1.6 \times 10^{-8}\ ohm - m \times 10^{6}\ A\,m^{-2}$

   $E = 1.6 \times 10^{-2}\ Vm^{-1}$.

5. The current , 6*x, entering at c gets divided equality into* $\frac{1}{2} \times 6x = = 2x$, *Along CA,CD and CE(fig) .at the junction A, the current 2x gets split into x and xalong AB and AF.*

   Applying kirchhoff's law to the path CABG, we get

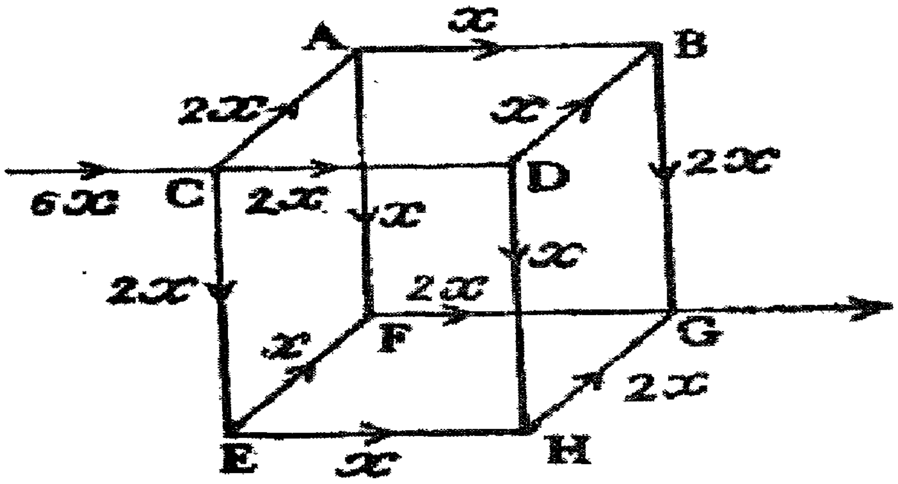
   $2x \times 2 + x \times 2 + 2x \times 2 = 10$

   $10x = 10$

   $x = 1$ A

   the current which enters at the node C =6x =6A.

   Equivalent resistance of the framework $= \frac{10}{6} = \frac{5}{3}$ *ohms*



2. 13 Suggested Readings:

3 Electricity and Magnetism - S Mahajan And A A Rangwala ,Tata McGrew Hill

4 Electricity and Magnetism - Dr.K.K.Tewari S.Chand & Co,2002.

5 Electricity and Magnetism with Electronics -D.N.Vasudeva S.Chand & Co,2002.

6 Electricity and Magnetism - Narayanamoorthy, Nagarathinam. 2nd Revised Edition

Unit III: Biot – Savat's Law

Structure:

3.1. Introduction
3.2. Objectives
3.3. Biot – Savart's law
    3.3.1 Biot – Savart's law & its applications:
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    3.5.1 Ampere's Law
    3.5.2 Differential Form of Ampere's Law
3.6. Magnetic field at the center of current carrying circular coil
3.7. Magnetic field due to Solenoid
3.8. Ballistic galvanometer (Theory and Damping corrections)
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    3.17.1 Coefficient of Coupling
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    3.17.3 Uses of Eddy Currents
3.18. Summary
3.19. Unit - end exercises
3.20. Problems for discussion
3.21. Answer to check your progress & Problems for discussion
3.22. Suggested Readings

3.1 Introduction

The concepts of electric current will be of much use to us and it is a vast area to study. However, for reasons of simplicity we confined our considerations of these concepts for charges that are placed in vacuum. For example, Coulomb's law of electrostatic force is the electric field due to a distribution of charges. And in this unit we are concentrate our attention on electric & magnetic field especially magnetic field due to solenoid, circular coil carrying current & the working principle and applications of ballistic galvanometer.

3.2 Objectives

After going through this unit you should able to

- understand what is meant by magnetic field, the right hand rule Biot-Savart law.
- define the magnetic field at a point in terms of the force on a steady current element and also an a moving charged particle.
- use Biot-Savart law to describe and compute the magnetic field generated by a simple current flow .
- find the magnetic field inside a long solenoid.
- compute the torque exerted by a steady magnetic field upon closed current loops.
- compute the magnetic field at the centre of a circular coil carrying current.
- appreciate how the forces on current – carrying conductors, placed in a magnetic field, are used to understand the working of galvanometer and motors.

3.3 The Biot-Savart Law

Consider a conductor XY carrying a current i (Fig 3.8.) Consider an element AB of length dl. O is the midpoint of AB. P is a point at a distance r from O. θ is the angle between dl and r.

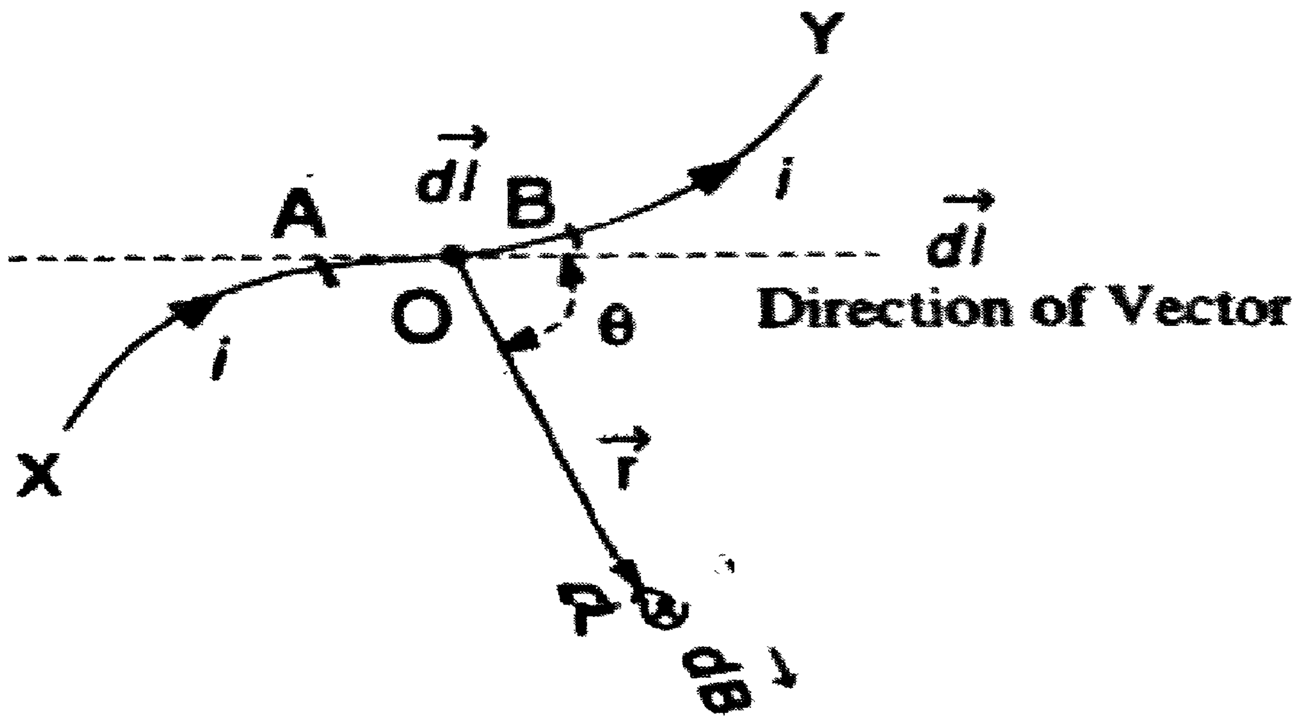


Fig 3.8

Magnetic induction dB at point P due to the current element dl is

$$dB = \left(\frac{\mu_0}{4\pi}\right)\frac{i\ (dl\ \text{x}\ \hat{r})}{r^2}$$

Here $\hat{r}$ is the unit vector along r.

The magnitude of dB is

$$dB = \left(\frac{\mu_0}{4\pi}\right)\frac{i\,(dl\,sin\theta)}{r^2} \qquad ..... (1)$$

$$dB \propto i$$
$$\propto dl$$
$$\propto sin\theta$$
$$\propto \frac{1}{r^2}$$

$$\frac{\mu_0}{4\pi} = 10^{-7} Wb\, A^{-1} m^{-1}$$

Here, μ_0 is called the permeability of free space.

Thus $\frac{\mu_0}{4\pi} = 10^{-7} Wb\, A^{-1} m^{-1}$

Eq. (1) is called **Biot – Savart's law.**

The direction of dB is that of the vector d*l* x r.

The total magnetic induction B at P due to the current flowing in entire length of the conductor is

$$B = \int dB = \left(\frac{\mu_0}{4\pi}\right)\int i\frac{dl \times \hat{r}}{r^2}$$

The vacuum, B is related to H (magnetic field intensity) by the formula

$$B = \mu_0 H$$

3.3.1 Biot – Savart's law & its applications
Magnetic Field of Electric Currents

Oersted's Experiment: Oersted first noticed the magnetic effect of electric current. He found that a pivoted magnetic needle gets deflected when a steady current is passed through a wire kept above or below and parallel to it (Fig 3.1). The deflection of the needle must be due to the magnetic field round the wire carrying current. The direction of deflection of the needle is given by Ampere's *Swimming rule.*

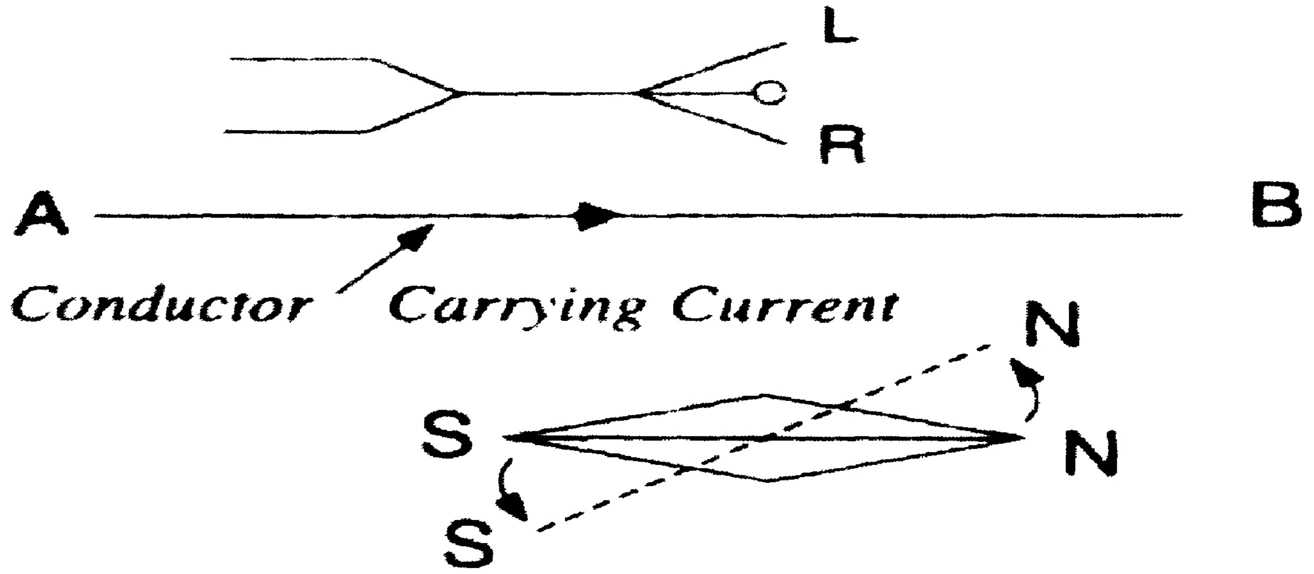


Fig 3.1

Imagine a man swimming along the wire in the direction of the current with his arms stretched and with his face towards the compass needle. The north pole of the needle is deflected towards his left hand.

Fig 3.2. Represents the direction of the magnetic lines of force due to a single straight conductor carrying current.

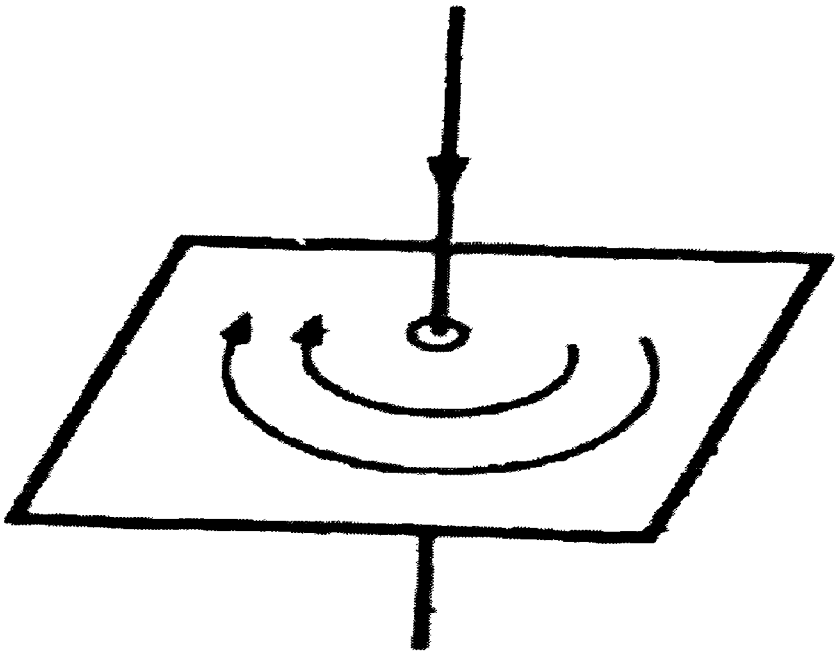


Fig 3.2

In a straight isolated conductor carrying current, the magnetic field is circular and concentric with the conductor. Its direction round the conductor is given by either Maxwell's cork screw rule or right hand clasp rule.

Maxwell's cork screw Rule: *If a right handed screw is turned to advance along the conductor in the direction of the current, the direction of rotation of the screw gives the direction of the lines of force* (Fig 3.3)

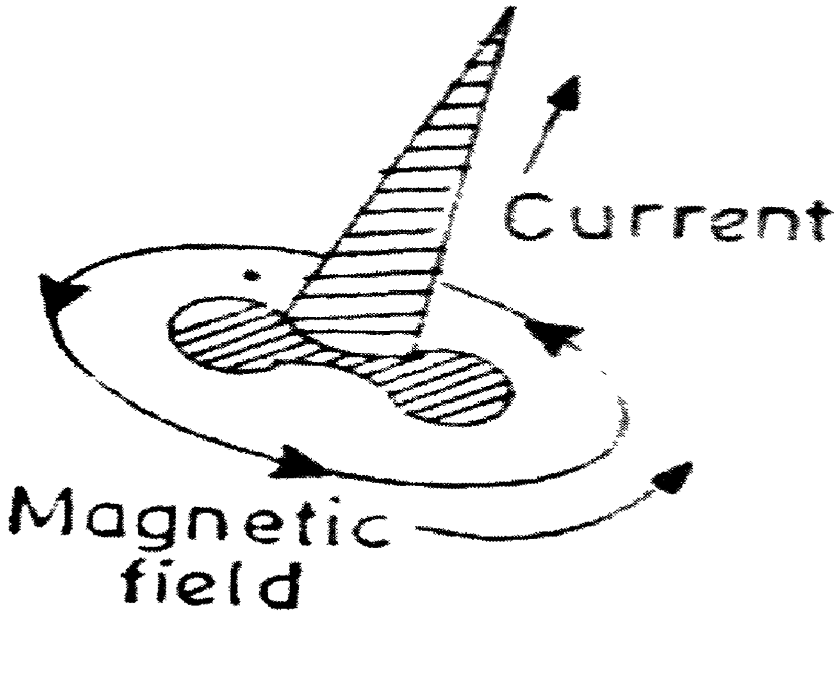


Fig 3.3

Right hand clasp Rule: *Clasp the conductor in the right hand with the thumb pointing in the direction of the current. Then the direction of bend of the rest of the fingers gives the direction of the magnetic lines of force* (Fig 3.4)

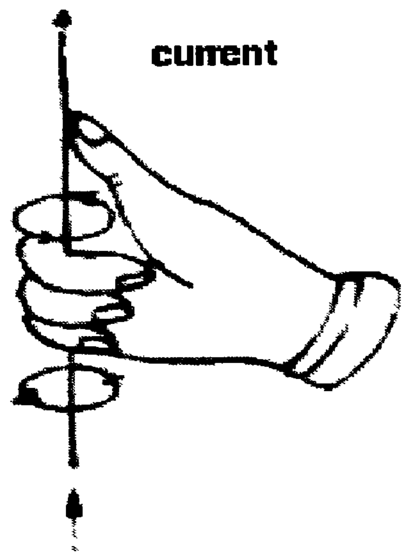


fig 3.4

The magnetic Field: The space around the current carrying conductor is defined as the site of a magnetic field. Magnetic field is a vector field whose magnitude and direction at any point are specified by a vector B called magnetic induction.
The definition of B: Consider a charge q moving with velocity v in a region where B exists (Fig 3.5). The charged particle experiences a force F in a direction perpendicular to both v and B.

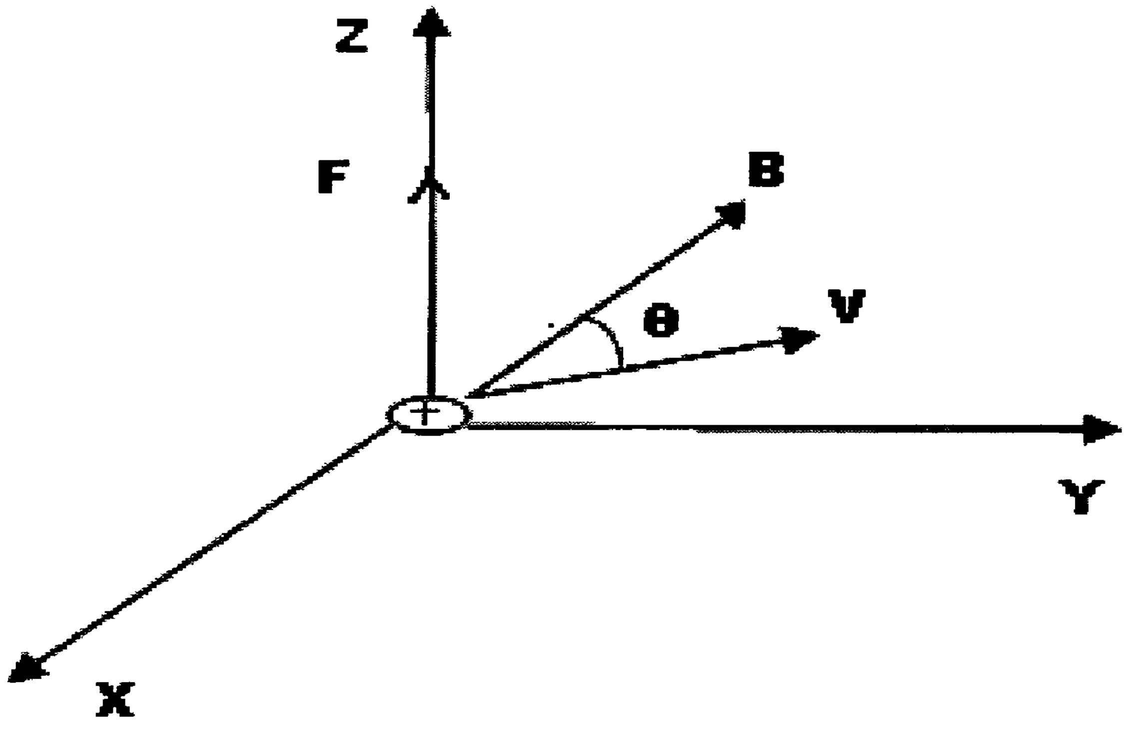


Fig 3.5

$F = q\,(\boldsymbol{v} \times B)$

$F = qBv\sin\theta$

Magnitude of $B = \frac{F}{q(v\sin\theta)}$

$$= \frac{\textit{Force on the charged particle}}{\textit{charge component of the velocity perpendicular to B.}}$$

The magnitude of the magnetic induction at a point is the ratio of the force on a moving charge at that point and the product of the charge and component of its velocity along a direction normal to the induction.

SI unit of B is Weber/metre^2 or tesla.

Magnetic induction B at a point is said to be one tesla if a charge of 1C moving with a velocity of 1 ms^{-1} at right angles to the magnetic induction field B at that point experiences a force of 1N.

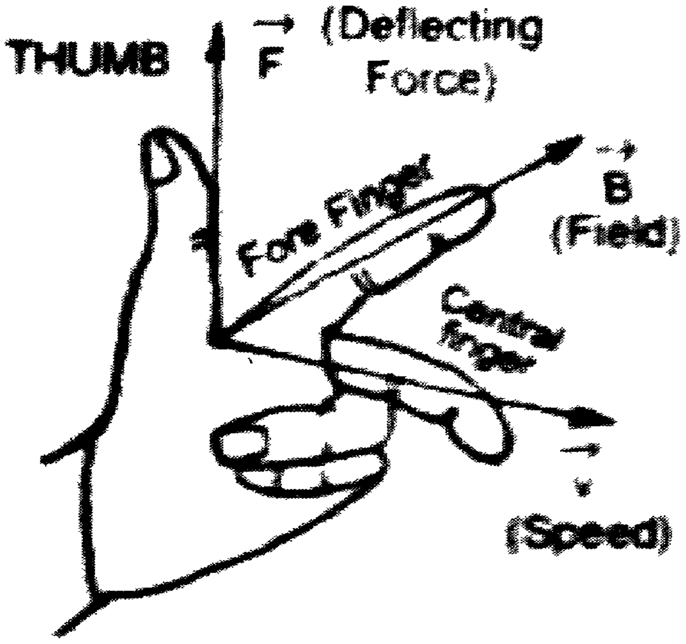


Fig 3.6

The direction of F is perpendicular to the plane containing v and B. It is given by Fleming's left hand rule.

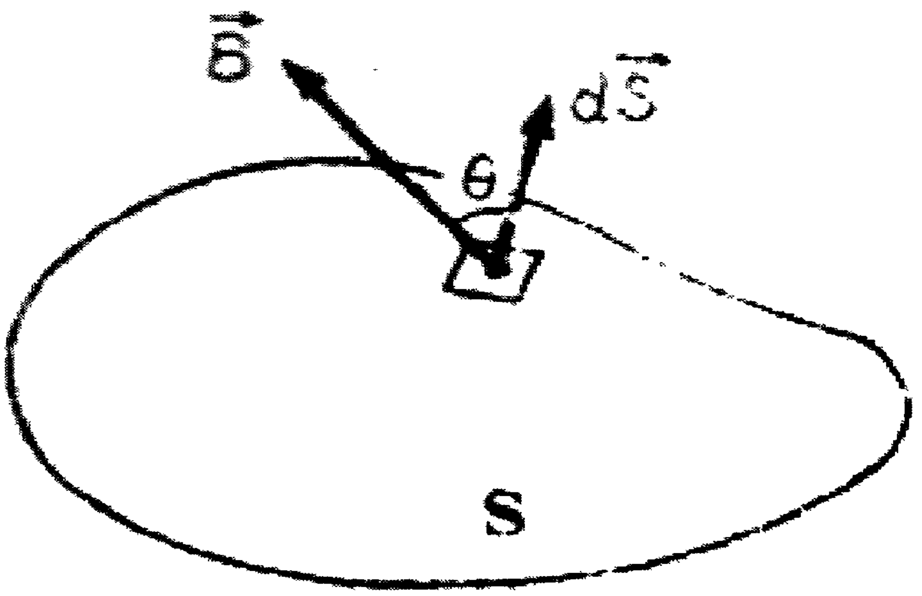


Fig 3.7

3.4 Long straight wire of infinite length

Magnetic induction at a Point due to Straight Conductor Carrying Current

Consider a straight conductor XY carrying a current i in the direction Y to X (Fig 3.9). P is a point at a perpendicular distance a from the conductor. Consider an element AB length dl. Let BP = r and < OBP = θ.

Magnetic induction at P due to the element AB = dB

$$= \left(\frac{\mu_0}{4\pi}\right)\frac{i\,dl\,sin\theta\,\hat{r}}{r^2} \quad .....(1)$$

From B, draw BC perpendicular to PA. Let $\angle$OPB = $\varphi, \angle BPA = d\varphi$.

Then, BC = dl sin θ= r d φ

$$dB = \left(\frac{\mu_0}{4\pi}\right)\frac{i\,rd\varphi}{r^2} = \left(\frac{\mu_0}{4\pi}\right)\frac{i\,d\varphi}{r}$$

In $\Delta\, OPB, \cos\varphi = \frac{a}{r}$ or $r = a/cos\varphi$

$$dB = \left(\frac{\mu_0}{4\pi}\right)\frac{icos\,\varphi d\varphi}{a} \quad ....(2)$$

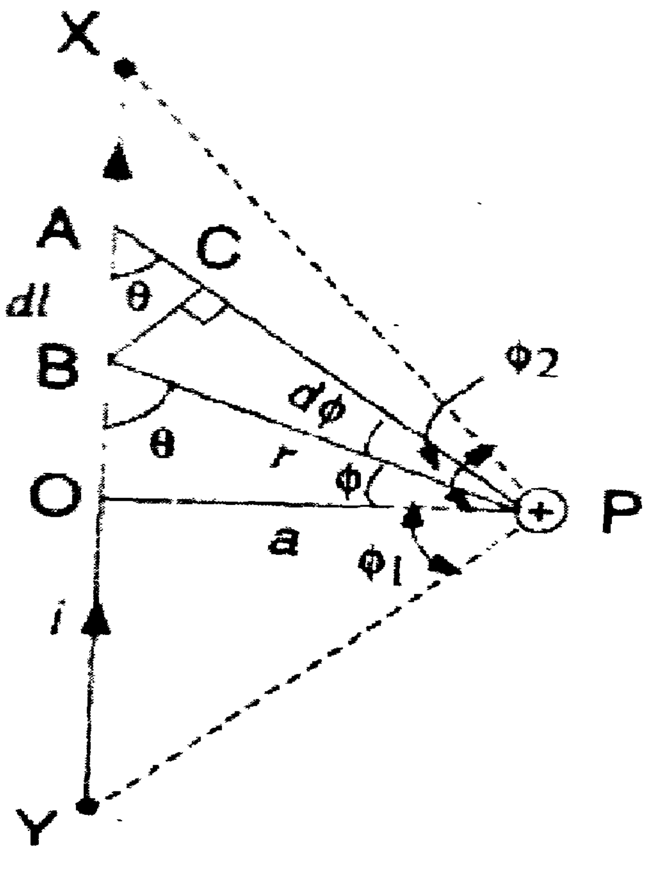


Fig 3.9

The direction of dB will be perpendicular to the plane containing dl and r. It will be directed into the page at P as shown by right hand rule.

Let φ_1 and φ_2 be the angles made by the ends of the wire at P. Then, magnetic induction at P due to the whole conductor is

$$B = \int_{-\varphi 1}^{\varphi 2} \left(\frac{\mu_0}{4\pi}\right)\frac{i\cos\varphi d\varphi}{a} = \left(\frac{\mu_0}{4\pi}\right)\frac{i}{a}[sin\varphi]_{-\varphi 1}^{\varphi 2}$$

$$= \left(\frac{\mu_0}{4\pi}\right)\frac{i}{a}[sin\ \varphi_2 - \sin\varphi_1]$$

$$= \left(\frac{\mu_0}{4\pi}\right)\frac{i}{a}[\sin\varphi_2 + \sin(\varphi_1)] \qquad ....(3)$$

If the conductor is infinitely long,

$$\varphi_1 = \varphi_2 = 90^o$$

$$B = \left(\frac{\mu_0}{4\pi}\right)\frac{i}{a}[1+1] = \frac{\mu_0 i}{2\pi a} \qquad .....(4)$$

Magnitude of B depends on i and a. $B \propto 1/a$

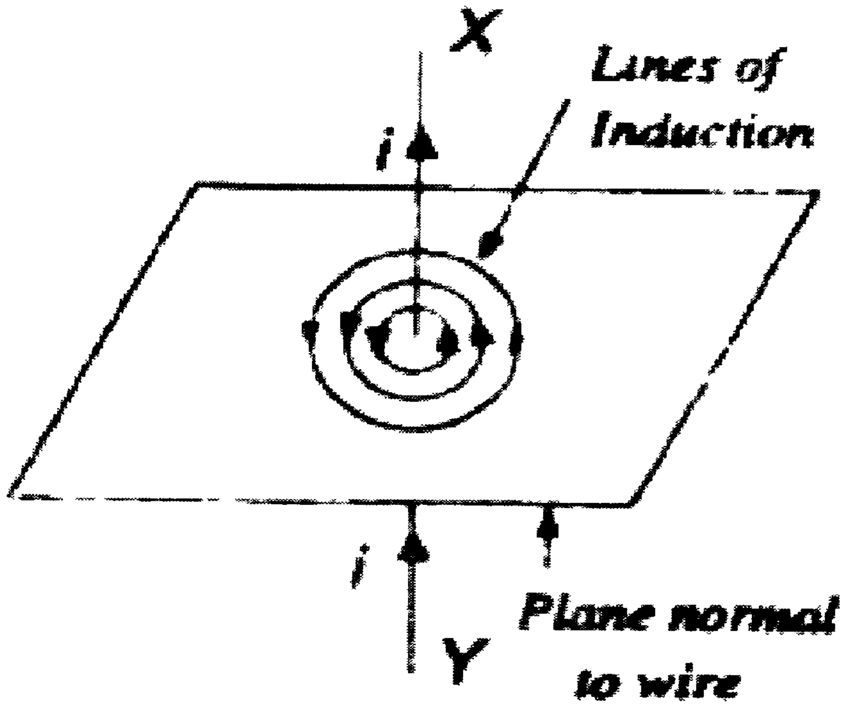


Fig 3.10

The lines of B form concentric circles around the wire (Fig 3.10)

Check your progress:
1. Find the magnetic induction at the centre of a square current loop of side 1 meter carrying a current of 1 ampere. Ans:-- -- -- --

Check your progress:
2. Of the three vectors in the equation **F = qv x B**, which pairs are always at right angles? Which may have any angle between them Ans:-- -- -- --

Check your progress:
3. Write one analogy and one difference between Coulomb's law and Biot-Savart law. Ans:-- -- -- --

3.5 Ampere's theorem

3.5.1 Ampere's Law

Statement: The line integral $\oint B.dl$ for a closed curve is equal to $\mu_0$times the net current i through the area bounded by the curve. That is,

$$\oint B.dl = \mu_0 i$$

Here $\mu_0$is the permeability constant

Proof. Consider a long straight conductor carrying a current it perpendicular to the page directed outward (Fig 3.11). According to Biot-Savart law, the magnetic of the magnetic induction at a distance r from its given by

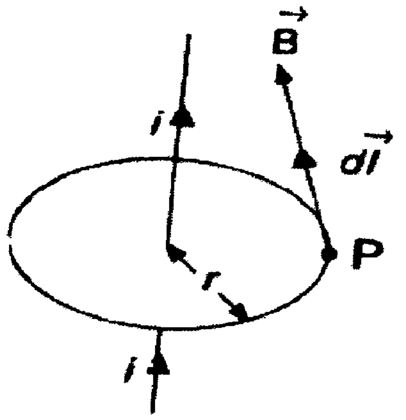


Fig 3.11

$$B = \frac{\mu_0 i}{2\pi r}$$

At each point on this circle, B has a constant magnitude B and dI which is always tangential to the path of integration, points in the same direction as B. Thus,

$$\oint B.dI = \oint B.dI = B \oint dI \ (B)(2\pi r)$$

Here $2\pi r = \oint dI$ is the circumference of the circle.

Substituting the value of B from Eq.(1), we gt

$\oint B.dI = \frac{\mu_0 i}{2\pi r}(2\pi r) = \mu_0 i$

Thus the integral $\oint B.dl\ is\ \mu_0$ times the current through the area bounded by the circle. This is ampere's law.

3.5.2 Differential Form of Ampere's Law

Let J be the current density in an element dS of the surface bounded by the closed path (Fig 3.12). Then,

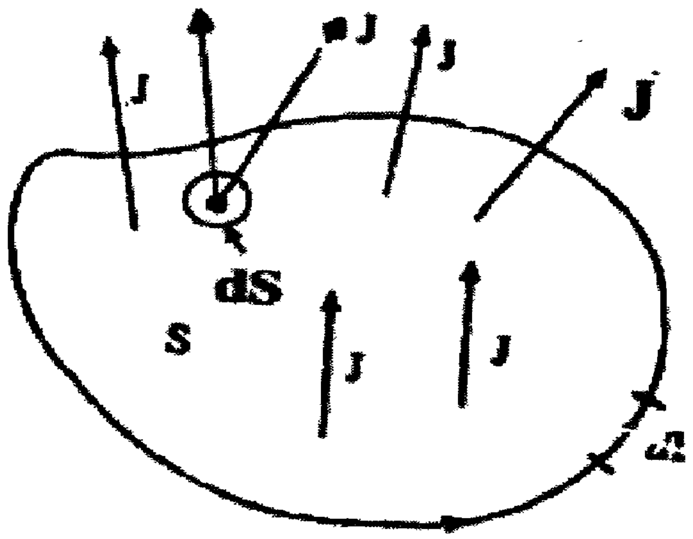


Fig 3.12

Total current i = $\int_s J.dS$

$$\oint B.dl = \mu_0 \int_s J.dS$$

Using Stokes theorem

$$\oint B.dl = \int_s curl\ B.dS$$

$$\int_s curl\ B.dS = \mu_0 \int_s J.dS$$

$$curl\ B = \mu_0 J$$

This is the differential form of Ampere's law.

3.6 Magnetic field at the center of current carrying circular coil

Consider a circular coil of one turn of radius r and centre O, carrying a current i. To find the flux density or the magnetic induction of the magnetic field produced at the centre, consider an element dl of the circular conductor (Fig 3.13)

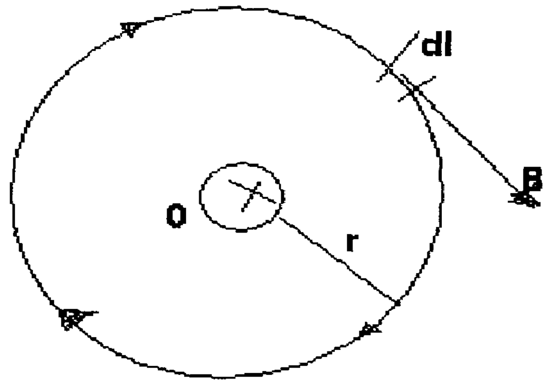


Fig 3.13

According to Biot-Savart's law.

$$dB = \left(\frac{\mu_0}{4\pi}\right)\frac{i\, dl \sin\theta}{r^2}$$

Hence , θ=90; sin 90°=1

$$dB = \frac{\mu_0}{4\pi}\frac{i\ dl}{r^2}$$

The whole circular conductor is made up of a large number of such elements of length dl.

Thus the total magnetic induction at the centre due to the entire coil is

$$B = \sum \frac{\mu_0}{4\pi}.\frac{i\ dl}{r^2} = \frac{\mu_0 i}{4\pi r^2}\sum dl$$

But $\sum dl = 2\pi r$.

$$B = \frac{\mu_0 i}{4\pi r^2}\ X\ 2\pi r$$

$$B = \frac{\mu_0 i}{2r}\frac{weber}{square\ meter}$$

If the circular coil has n turns of mean radius r, the magnetic induction of the field produced at the centre is given by

$$B = \frac{n\mu_0 i}{2r}$$

Here B is in tesla, r is in metres, and i is in amperes.

3.7 Magnetic field due to Solenoid

Let L represents the length of the solenoid and N the total number of turns in its winding (Fig 3.14). The number of turns per unit length is then N/L.a is the radius of the solenoid. A current i is flowing in the solenoid. The solenoid

contains air in its core. Let us find the magnetic induction B at a point P on the axis of the solenoid.

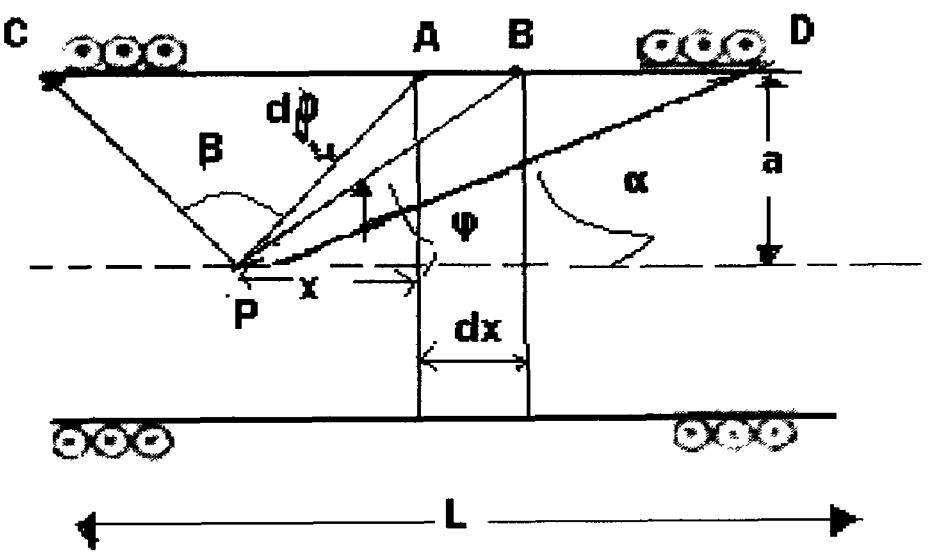


Fig 3.14

Consider an elementary length dx of the solenoid, at a distance x from P. We can regard this element AB as a circular coil of radius a containing N dx/L turns.

Magnitude induction at P due to the element dx is

$$dB = \frac{\mu_0 i a^2}{2} \cdot \frac{N\,dx}{L} \cdot \frac{1}{(a^2 + x^2)^{\frac{3}{2}}}$$

Let us use the angle φ instead of x as the independent variable.

Then , X=a cotφ; dx= -a cosec$^2\varphi$dφ

Substituting these values of x and dx in Eq.(1), we get

$$dB = \frac{\mu_0 i a^2}{2} \cdot \frac{N}{L} \cdot \frac{a cosec^2\,\varphi d\varphi}{(a^2 + a^2\,cot^2 \varphi)^{\frac{3}{2}}} = -\frac{\mu_0 i N}{2L} \sin\varphi\, d\varphi$$

The magnetic induction at P due to the entire length of the solenoid is

$$B = -\frac{\mu_0 i N}{2L} \int_{\beta}^{\alpha} \sin\varphi\, d\varphi$$

$$= -\frac{\mu_0 i N}{2L}[cos\alpha - \cos\beta]$$

The direction of B is parallel to the axis of the solenoid.

Note. If the core of the solenoid consists of magnetic material of permeability μ, the magnetic induction inside such a solenoid is

$$B = \frac{\mu_0 i N}{2L}[cos\alpha - \cos\beta]$$

Where $\mu = \mu_0 \mu_r and\ \mu = B/H$

Special cases

(i) At a point well inside a very long solenoid: $\alpha = 0, \beta = 180^o$

$$B = \mu_0 iN/L$$

(ii) At an axial point at one end of a long solenoid: $\alpha = 0, \beta = 90^o$

$$B = \mu_0 iN/2L$$

Hence the magnetic induction at either end is one-half its magnitude at points well inside the solenoid.

Magnetic Field inside a long solenoid.

Consider a long straight solenoid having n turns per unit length. Let i be the current flowing in the solenoid. It is experimentally noted that magnetic field outside the solenoid is very small in comparison with the field inside. The lines of induction inside the solenoid are straight and parallel (Fig 3.15)

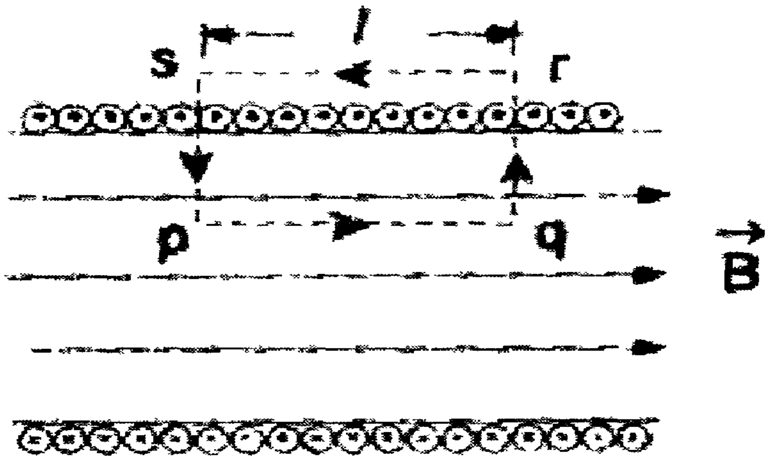


Fig 3.15

Consider a closed path pqrs. The line integral of magnetic field B along path pqrs is

$$\oint_{pqrs} B.dl = \int_{pq} B.dl + \int_{qr} B.dl + \int_{rs} B.dl + \int_{sp} B.dl$$

Let pq = l for path pq, B and dl are along same direction.

$$\int_{pq} B.dl = \int B\,dl = Bl$$

For paths qr and sp, B and dl are mutually perpendicular

$$\int_{qr} B.dl = \int_{sp} B.dl = \int B\,dl\cos 90^o = 0$$

For path rs , B = 0 (since field is zero outside a solenoid)

$$\int_{rs} B.dl = 0$$

Eq (1) becomes $\oint_{pqrs} B.dl = \int_{pq} B.dl = Bl$

By Amperes's law, $\oint_{pqrs} B.dl = \mu_0$ x net current enclosed by path

$Bl = \mu_0 (nl)\, i$

$B = \mu_0\, ni$

3.8 Ballistic Galvanometer

Principle. When a current is passed though a coil, suspended freely in a magnetic field, it experiences a force in a direction given by Fleming's left hand rule.

Construction. It consists of rectangular coil of thin copper wire wound on a non-metallic frame of ivory (Fig 3.16). It is suspended by means of a phosphor bronze wire between the poles of a powerful horse shoe magnet. A small circular mirror is attached to the suspension wire. Lower end of the coil is connected to a hair-spring. The upper end of the suspension wire and the lower end of the spring are connected to terminals T_1 and T_2. A cylindrical soft iron core (C)is placed symmetrically inside the coil between the magnetic poles which are also made cylindrical in shape. This iron core concentrates the magnetic field and helps in producing radial field The B.G is used to measure electric charge. The charge has to pass through the coil as quickly as possible and before the coil starts moving. The coil thus gets an impulse and a throw is registered. To achieve this result, a coil of high moment of inertia is used so that the period of oscillation of the coil is fairly large. The oscillations of the coil are practically undamped.

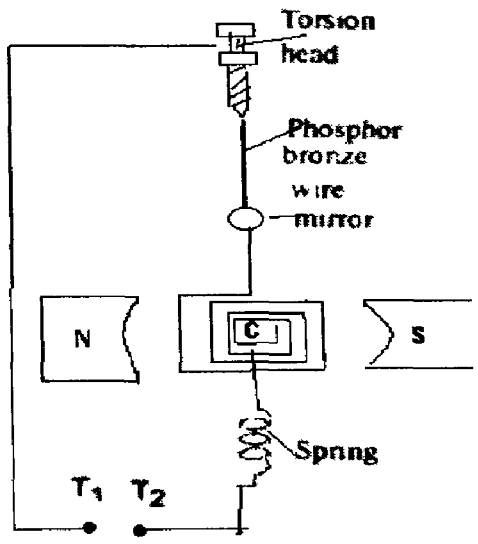


Fig 3.16

Theory (i) Consider a rectangular coil of N turns placed in a uniform magnetic field of magnetic induction B (Fig 3.17.) Let l be the length of the coil and b its breath.

Area of the coil = A = lb

When a current i passes through the coil,

torque on the coil = τ = NiBA ....(1)

If the current passes for a short interval dt, the angular impulse produced in the coil is

τ dt = NiBA dt. ....(2)

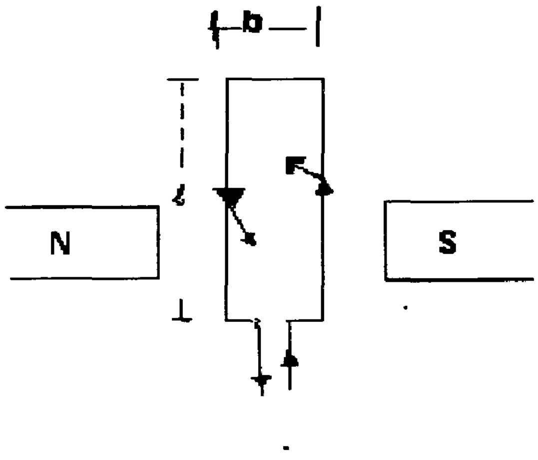


Fig 3.17

If the current passes for t seconds, the total angular impulse given to the coil is

$\int_0^1 \tau dt = NBA \int_0^1 i\, dt = NBA\, q$.................(3)

Here $\int_0^1 i\, dt = q$ = total charge pasing through the galvanometer coil.

Let I be the moment of inertia of the coil about the axis of suspension and ω its angular velocity.

Change in angular momentum of the coil = I ω.................(4)

$I\omega = NBA\, q$.................(5)

(ii)The kinetic energy of the moving system $\frac{1}{2}I\omega^2$ is used in twisting the suspension wire through an angle θ. Let c be the restoring torque per unit twist of the suspension wire.

Work done in twisting the suspension wire by an angle $\theta = \frac{1}{2}c\theta^2$

$\frac{1}{2}I\omega^2 = \frac{1}{2}c\theta^2$ ....(6)

or $I\omega^2 = c\theta^2$

(iii) The period of oscillation of the coil is

$$T = 2\pi\sqrt{\left(\frac{I}{c}\right)} \quad or \quad T^2 = \frac{4\pi^2 I}{c}$$

$$I = \frac{T^2 c}{4\pi^2} \quad\text{.................(7)}$$

Multiplying Eqs.(6) and (7) $I^2\omega^2 = \frac{c^2T^2\theta^2}{4\pi^2}$

or $\quad I\omega = \frac{cT\theta}{2\pi}$ ....(8)

equating (5) and (8) NBAq $=\frac{cT\theta}{2\pi}$

or $\quad q = \left(\frac{T}{2\pi}\right)\left(\frac{c}{NBA}\right)\theta\ \omega$ ....(9)

This gives the relation between the charge flowing and the ballistic throw θ of the galvanometer .q$\propto \theta$.

$\left(\frac{T}{2\pi}\right)\left(\frac{C}{NBA}\right)$ *is called the ballistic reduction factor* (K)

q=Kθ.................(10)

Correction for Damping in ballistic Galvanometer

We have assumed that the whole of the kinetic energy imparted to the coil is used in twisting the suspension of the coil. In actual practice, the motion of the coil is damped by air resistance and the induced current produced in the coil. The first throw of the galvanometer is, therefore, smaller than it would have been in the absence of damping. The correct value of first throw is however obtained by applying damping correction.

Let $\theta_1, \theta_2, \theta_3, \ldots\ldots$ be the successive maximum deflections from zero position to the right and left (Fig 4.29) Then it is found that

$$\frac{\theta_1}{\theta_2} = \frac{\theta_2}{\theta_3} = \frac{\theta_3}{\theta_4} = \cdots = d \quad\text{.................(1)}$$

The constant d is called the decrement per half vibration

Let d = e^{λ} *so that* $\lambda = \log_e d$

Here λ is called the logarithmic decrement

$\frac{\theta_1}{\theta_3} = \frac{\theta_1}{\theta_2} = \frac{\theta_2}{\theta_3} = d^2 = e^{2\lambda}$

Let θ be the true first throw in the absence of damping

$\theta > \theta_1$. The first throw θ_1 is observed after the coil completes a quarter of vibration. In this case, the value of the decrement would be $e^{\lambda/2}$

$$\frac{\theta}{\theta_1} = e^{\lambda/2} \approx \left(1 + \frac{\lambda}{2}\right)$$

or $$\theta = \theta_1\left(1 + \frac{\lambda}{2}\right) \quad \text{.................(2)}$$

we can calculate λ by observing the first throw θ_1 and the eleventh throw θ_{11}.

$$\frac{\theta_1}{\theta_{11}} = \frac{\theta_1}{\theta_2}\cdot\frac{\theta_2}{\theta_3}\cdot\frac{\theta_3}{\theta_4}\cdot\frac{\theta_4}{\theta_5}\cdot\frac{\theta_5}{\theta_6}\cdot\frac{\theta_6}{\theta_7}\cdot\frac{\theta_7}{\theta_8}\cdot\frac{\theta_8}{\theta_9}\cdot\frac{\theta_9}{\theta_{10}}\cdot\frac{\theta_{10}}{\theta_{11}} = e^{10\lambda}$$

or $$\lambda = \frac{1}{10}log_e\left(\frac{\theta_1}{\theta_{11}}\right) = \frac{2.3026}{10}log_e\left(\frac{\theta_1}{\theta_{11}}\right) \quad \text{.................(3)}$$

$$q = \left(\frac{T}{2\pi}\right)\left(\frac{C}{NBA}\right)\theta_1\left(1 + \frac{\lambda}{2}\right) \quad \text{.................(4)}$$

3.9 Dead – beat and Ballistic Galvanometers

Galvanometers are classified as (i) Dead-beat or aperiodic and (ii) Ballistic galvanometers

A moving coil galvanometer is which the coil is wound on a metallic conducting frame is known as a dead-beat galvanometer. It is called "dead-beat" because it gives a steady deflection without producing any oscillation, when a steady current is passed through the coil

3.9.1 Conditions for a moving coil galvanometer to be dead beat

(i) Moment of inertia of the system should be small
(ii) Coil should be mounted on a conducting frame
(iii) Suspension fiber should be comparatively thicker

3.9.2 Conditions for a moving coil galvanometer to be ballistic

(i) The moment of inertia of moving system should be large.
(ii) Suspension fibre should be very fine
(iii) Air resistance should be small
(iv) The damping should be small i.e., the coil should be wound on a non-conducting frame.

3.9.3 Measurement of charge sensitiveness (figure of merit of a B.G)

The charge passing through a B.G. is given by

$$q = \left(\frac{T}{2\pi}\right)\left(\frac{C}{NBA}\right)\theta_1\left(1 + \frac{\lambda}{2}\right) = K\theta_1\left(1 + \frac{\lambda}{2}\right)$$

Here K is charge sensitiveness of figure of merit of the galvanometer. It is also known as the 'ballistic reduction factor' of the galvanometer.

The charge that should circulate through the coil to produce an undamped throw of 1 mm in the spot of light on a scale placed at distance of 1 metre from the mirror is called the charge sensitiveness K of the ballistic galvanometer.

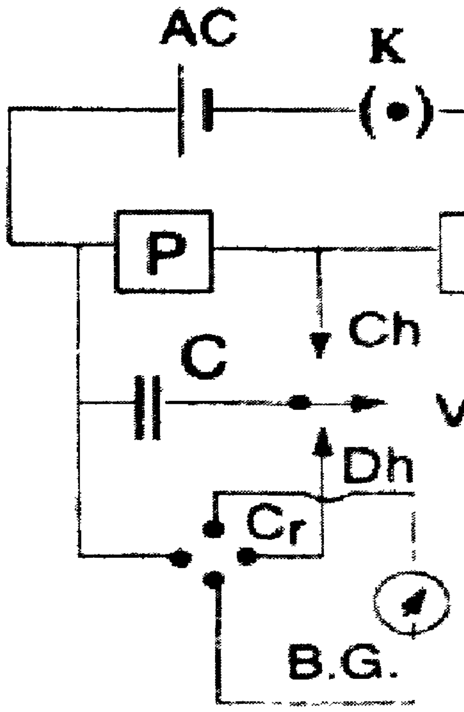


Fig 3.18

Two resistance boxes P and Q and a key K are connected in series with an accumulator of emf E (fig 3.18). A capacitor of known capacitance C is connected to P through the vibrator V and charging terminal Ch of the charge-discharge key. The capacitor is charged with the p.d. across P. The charge on the capacitor can be discharged through the B.G., included in the circuit through the vibrator and discharge terminal of the charge – discharge key. A commutator Cr is included in the circuit to reverse the charge in the B.G.

1000Ω in P and 9000Ω in Q are included. The capacitor is charged and immediately discharged through the B.G. The first throw θ_1 is noted. The experiment is repeated with P = 2000Ω, 3000Ω etc., keeping (P+Q) = 10,000Ω. Mean value of P/θ_1 is calculated.

Let the capacitance of the capacitor be C μF.

Charge on the capacitor $q = \frac{EP}{(P+Q)}C$ μc

This charge produces a throw θ_1

Undamped throw $\theta = \theta_1\left(1+\frac{\lambda}{2}\right)$

Charge required to produce unit deflection = K

$$K = \theta_1\left(1+\frac{\lambda}{2}\right) = \frac{EP}{(P+Q)} \text{ x } C$$

$$K = \frac{EC}{(P+Q)} \text{ x } \frac{P}{\theta_1\left(1+\frac{1}{2}\lambda\right)} \quad \mu C \,/\, div$$

The value of $\lambda$is obtained by observing the first throw θ_1 and then the eleventh throw θ_{11} and using the relation.

$$\lambda = \frac{1}{10} log_e \left(\frac{\theta_1}{\theta_{11}}\right) = \frac{1}{10} \text{ x } 2.3026 \text{ x } log_{10} \left(\frac{\theta_1}{\theta_{11}}\right)$$

3.9.4 Difference between Dead –Beat and Ballistic Galvanometers

Dead – beat Galvanometer	Ballistic Galvanometer
1. It measure steady current	1. It measures charge
2. The steady deflection measures the current.	2. The throw measures the charge
3. The coil is wound on a metal frame to increase electromagnetic damping	3. The coil is wound on a non-metallic frame to reduce electromagnetic damping
4. The coil is non-oscillatory due to large damping	4. The coil is oscillatory due to small damping
5. The coil rotates due to constant torque.	5. The momentary passage of charge causes impulse on the coil. The torque is zero when the coil rotates.

USES OF BALLISTIC GALVANOMETER

3.10 Determination of absolute capacity of a conductor

Absolute Capacitance of a capacitor

(i) Two resistance boxes P and Q are connected in series with an accumulator of emf E (Fig 3.19).

A small resistance ($\approx 0.1\Omega$) is taken in P and a large resistance (9999.9Ω) in Q so that $P + Q = 10{,}000\Omega$. The galvanometer (MG) and a resistance box R are connected across P.

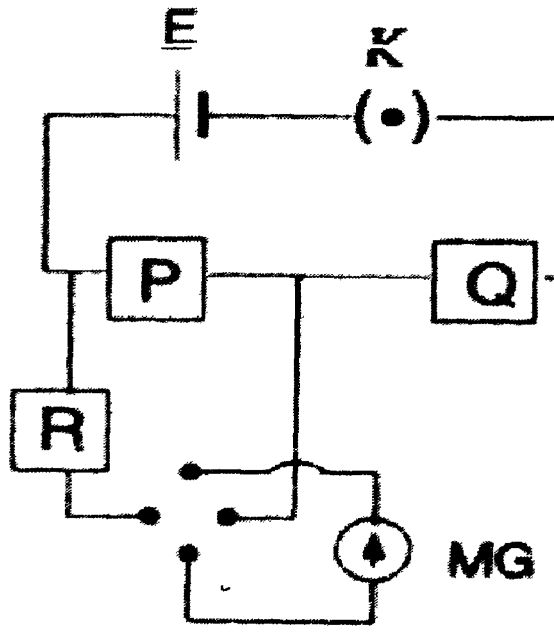


Fig 3.19

With no resistance in R, the steady deflection d of the galvanometer is found. A suitable resistance is taken in R till the deflection becomes half. The resistance in R is the galvanometer resistance R_g. The experiment is repeated for various values of P keeping P + Q constant.

Current through galvanometer $= \frac{EP}{P+Q} \times \frac{1}{R_g}$ ....(1)

Current through galvanometer is also $= \frac{c}{BAN} d$................(2)

From Eqns. (1) and (2)

$$\frac{c}{BAN} d = \frac{EP}{P+Q} \times \frac{1}{R_g}$$

$$\frac{c}{BAN} d = \frac{EP}{P+Q} \left(\frac{P}{d}\right) \frac{1}{R_g} ....(3)$$

The mean value of P/d is found out from this part of the experiment.

(ii) The galvanometer coil is set oscillating freely in open circuit. The time for 10 oscillations is found and the period T is calculated.

(iii)Connections are made as shown in Fig.3.20. Resistances P_1 (1000Ω) and Q_1 (9000Ω) are included in the boxes P and Q respectively.

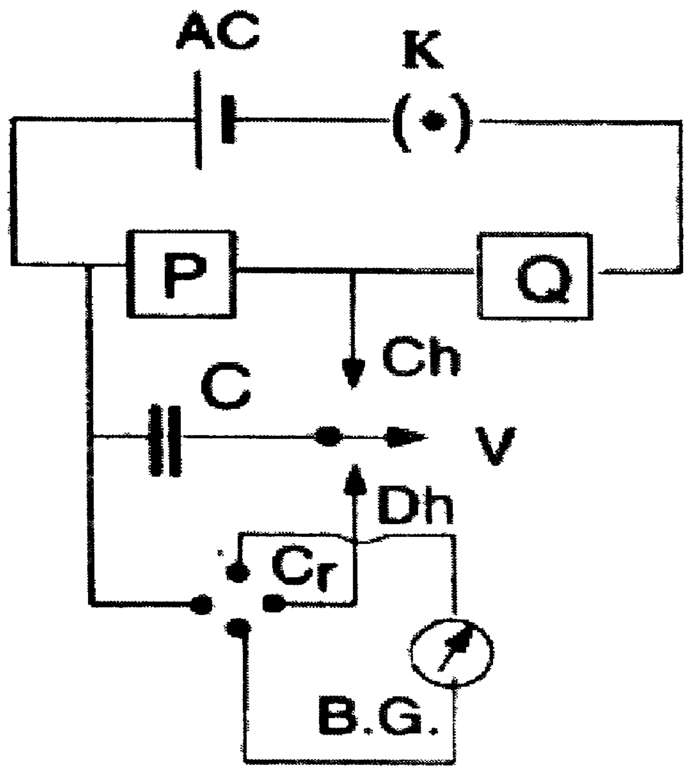


Fig 3.20

Potential difference across $P_1 = V = \frac{EP_1}{(P_1+Q_1)}$

The drop of potential across P_1 is used to charge the capacitor, by connecting the terminals Ch and V of the charge – discharge key.

Charge on the capacitor =q=CV = C x $\frac{EP_1}{(P_1+Q_1)}$................(4)

The terminals Dh and V are now connected so that the acitor gets discharged through the galvanometer. The first throw, is need

$$q = \frac{T}{2\pi}\frac{c}{BAN}\theta_1\left(1+\frac{1}{2}\lambda\right)\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

$$= C \times \frac{EP_1}{(P_1+Q_1)} = \frac{T}{2\pi}\frac{c}{BAN}\theta_1\left(1+\frac{1}{2}\lambda\right)$$

[From Eqns.(4) and (5)]

or $$C = \frac{T}{2\pi}\frac{c}{BAN}\left(\frac{\theta_1}{P_1}\right)\frac{P_1+Q_1}{E}\left(1+\frac{1}{2}\lambda\right)\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

substituting the value of (c/BAN) from Eq (3) in Eq.(6)

$$C = \frac{T}{2\pi}\frac{E}{P+Q}\left(\frac{P}{d}\right)\frac{1}{R_g}\left(\frac{Q_1}{P_1}\right)\frac{P_1+Q_1}{E}\left(1+\frac{1}{2}\lambda\right)$$

But $P + Q = P_1 + Q_1$

$$C = \frac{T}{2\pi}\frac{1}{R_g}\left(\frac{P}{d}\right)\left(\frac{Q_1}{P_1}\right)\left(1+\frac{1}{2}\lambda\right)\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

The experiment is repeated for various values of P_1 keeping $(P_1 + Q_1)$ the same as $P + Q$. The mean value of $\frac{\theta_1}{P_1}$ is calculated.

(iv)To find λ, the coil is set oscillating. The first throw θ_1 and the eleventh throw θ_{11} are noted. Then,

$$\lambda = \frac{2.3026}{10}\log_{10}\frac{\theta_1}{\theta_{11}}$$

Substituting the values of T, R_g, (P/d), $(\frac{\theta_1}{P_1})$ and λ in Eq.(7), C (the value of capacitance of the given capacitor) is determined.

3.11 Comparison of Capacitances using B.G

Connections are made as shown in Fig.3.21.Let C_1 and C_2 be the capacitances of the two given capacitors. These capacitors are connected to the end terminals of the DPDT key. A resistance of 1000Ω is introduced in P and 9000Ω in Q.

The capacitor C_1 is charged to the p.d. across P. the charge on C_1 is then discharged through the B.G. The throws in the B.G. are noted before and after reversing the commutator. The mean throwθ_1 is found out.

With the same resistance in P and Q, the handle of the DPDT key is thrown on the side of C_2, C_2 is charged to the same potential across P. The charge on

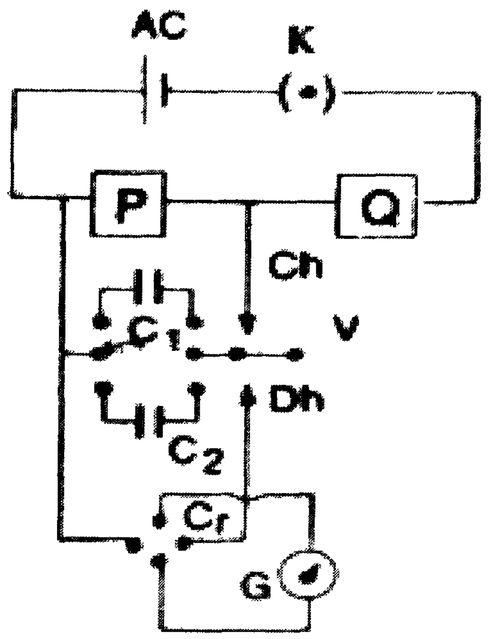


Fig 3.21

C_2 is then discharged through the B.G. The mean throw θ_2 is fund out.

Let V be the p.d. across the terminals of P. Then

In the first case . $q_1 = C_1 \text{x} V = K\theta_1\left(1+\frac{1}{2}\lambda\right)$

In the second case. $q_2 = C_2 \text{x} V = K\theta_2\left(1+\frac{1}{2}\lambda\right)$

$$\frac{C_1}{C_2} = \frac{\theta_1}{\theta_2}$$

The experiment is repeated for different values of P keeping (P+Q) constant.

P + Q ohms	P ohms	Throw due to C_1			Throw due to C_2			$\frac{C_1}{C_2} = \frac{\theta_1}{\theta_2}$
		Left	Right	Mean θ_1	left	Right	Mean θ_2	

Mean $\frac{C_1}{C_2} = \frac{\theta_1}{\theta_2}$

Check your progress:
4. What is the ratio of the charge sensitivity and current sensitivity of a ballistic galvanometer? Ans:-- -- -- ---------------

Check your progress:

5. Write down the differences between Dead-beat and ballistic galvanometers.

   Ans:--

Check your progress:

6. A circular loop of radius 5.0cm consists of 10 turns of wire. A current of 3.0 A flows in the wire. What is the magnitude of the loop's magnetic moment? Suppose initially the magnetic moment is aligned with a uniform magnetic field of 100 Gauss. Now the loop is turned 90° from its original orientation. How much torque is required to hold the loop in its new orientations?

   Ans:--

3.12 Faraday's Laws of Electro Magnetic Induction &Lenz's law

Whenever magnetic lines of force are cut by a closed circuit, an induced current flows in the circuit. The current lasts only so long as the flux is changing. The emf which produces this current, is called induced emf. This phenomenon is called electromagnetic induction.

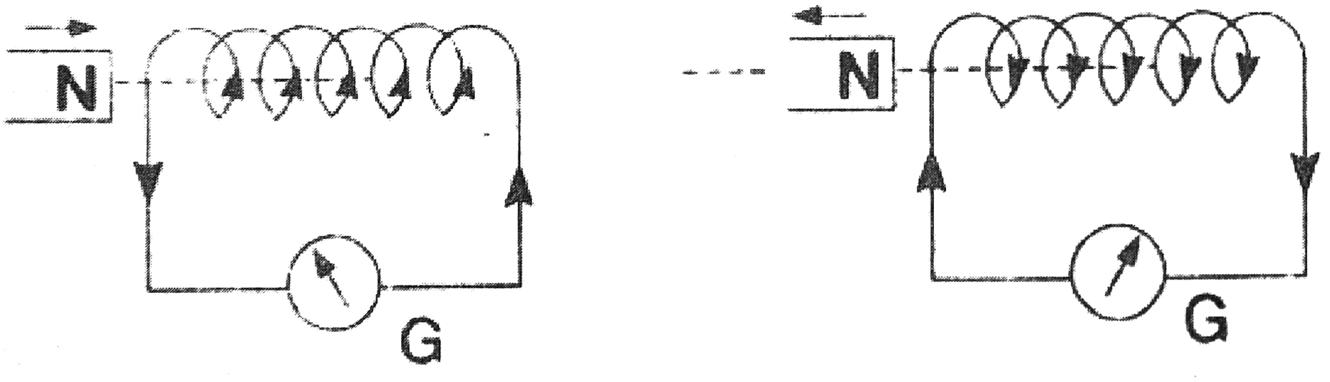


Fig 3.22

(i) When the magnet is moved towards the coil, with its north pole facing the coil, the galvanometer shows a deflection in one direction.

(ii) When the magnet is moved away from the coil, the galvanometer shows a deflection in the opposite direction.

(iii) If the experiment is repeated with south pole of the magnet facing the coil, the deflections in the galvanometer are reversed.

(iv) When the magnet is stationary, there is no deflection in the galvanometer.

(v) It is further observed that the deflection increases with the velocity of the magnet relative to the coil

(vi) The same results are obtained if the magnet is kept fixed and coil moved.

Experiment: Fig 3.23 shows a primary coil P connected to a battery and tap key K, and a secondary coil connected to a galvanometer.

(i) When the battery circuit is closed by pressing K and then broken, the galvanometer shows a deflection first in one direction and then in the other direction.

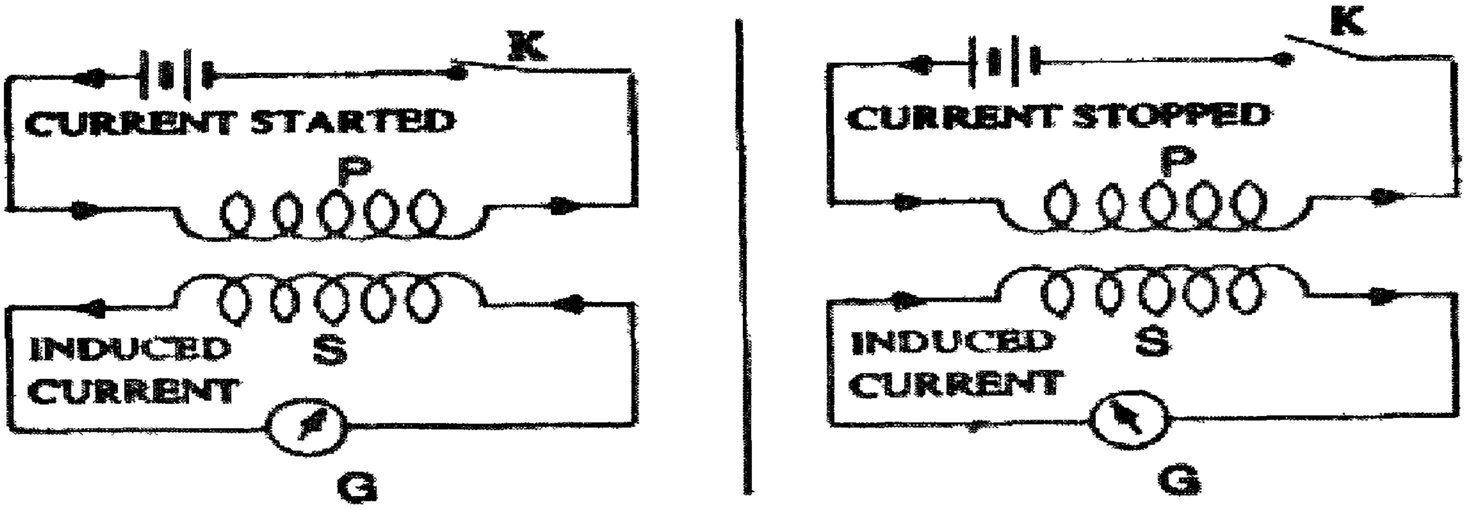


Fig 3.23

(ii) If the current in the primary flows continuously, no deflection is produced in the galvanometer. The deflection is produced only at the time of make and break.

Similar effects are observed while increasing or decreasing the primary current or changing the relative position of the coils.

From his experimental results, Faraday gave two laws:

(i) Whenever the magnetic flux through a conductor is changed, an emf is induced in the conductor. The magnitude of the induced emf is equal to the rate of change of magnetic flux through the circuit.

If φ is the magnetic flux linked with the circuit at any instant t and ε is the induced e.m.f. then

$$\varepsilon \propto \frac{d\varphi}{dt}$$

(ii) The direction of the induced emf, or current, is such as to oppose the change that produced it.

This is also known as **Lenz's law.**

Combining both laws, $\varepsilon = -\frac{d\varphi}{dt}$

3.13 Self – Induction

When a current flows in a coil, a magnetic field is set up in it. (Fig 3.24.a). If the current through the coil is changed, the flux linked with the coil also changes. An induced emf is set up in the coil. By Lenz's law, the direction of induced emf is such as to oppose the change in current. When the current is increasing, the induced emf is against the current (Fig 3.24.b). When the current is decreasing, the induced emf is in the direction of current (Fig 3.24.c). The phenomenon is called self-induction. The induced emf is called back emf.

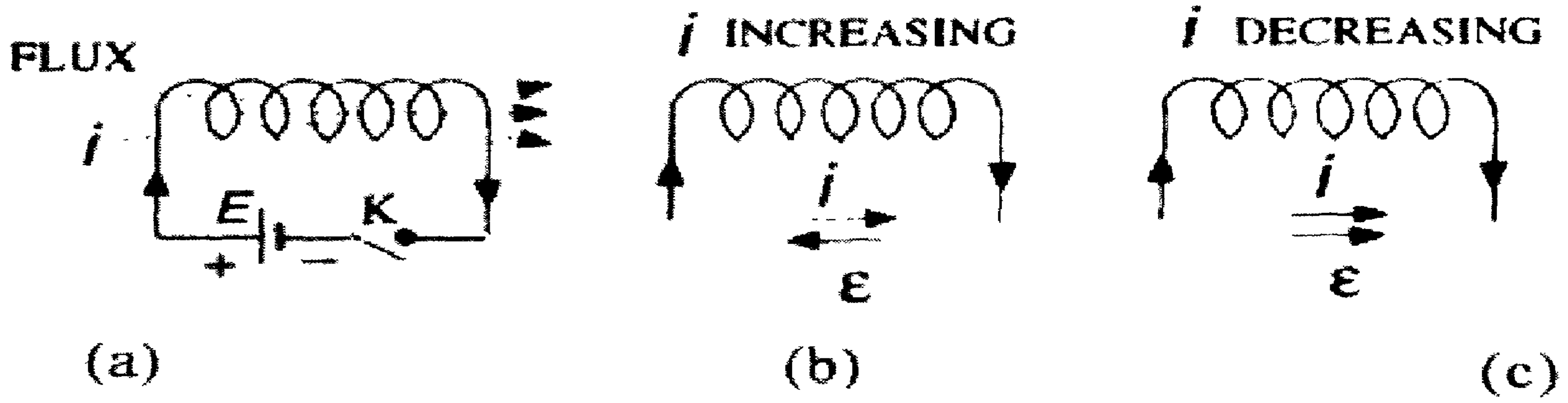
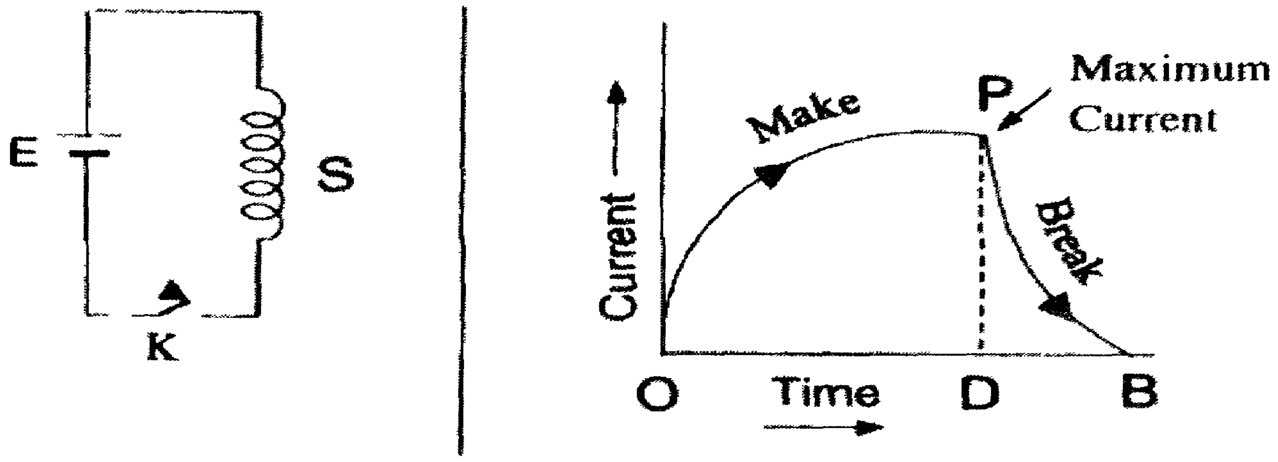


Fig 3.24.

When the current in a coil is switched on, self-induction opposes the growth of the current. Hence the current increases slowly and takes some time = OD to increases from zero to maximum value (Fig 3.25) During the period of growth, energy is absorbed. When the current is switched off, self – induction opposes the decay of current. So the current does not become zero instantaneously but takes some time = DB. When the current is switched, off the stored energy is given back in the form of spark. The effect of self-induction in an electric circuit is similar to inertia in motion.



(a) (b) Fig 3.25

The phenomenon of the production of an induced e.m.f. in a circuit itself due to the change incurrent through it is called self induction and the induced e.m.f. is called back e.m.f.

Self Inductance: The magnetic flux φ produced in a coil is directly proportional to the current I flowing in the coil

$$\varphi \; \alpha \; I$$

or

$$\varphi = L\, I \ldots\ldots\ldots (1)$$

Here L is a constant of proportionality, called the coefficient of self induction or self inductance of the coil

From Eq.(1) if I = 1 , then $\varphi = L$

Therefore , the self – inductance of a coil is the total magnetic flux linked with it when a unit current passes through it

When the flux changes, the back e.m.f. induced in the coil is given by (Faraday's law)

$$\varepsilon = -\frac{d\varphi}{dt} = -L\frac{dI}{dt} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

If $\frac{dI}{dt} = 1\, unit\, , |\varepsilon| = L$

Self – inductance of a coil is numerically equal to the induced emf when current in it is changing at unit rate.

S.I unit of self-inductance is henry (H)

From Eq.(1) one henry is the self-inductance of a coil or circuit, if a current of one ampere produces a magnetic flux of one weber in it.

From Eq.(2) one henry is the self-inductance of a coil or circuit, if an induced e.m.f. of one volt is produced in it due to a rate of change of current of one ampere per sec.

3.14 Energy Stored in an Inductor

Consider an electric circuit containing inductance. When the circuit is closed, a back e.m.f. is induced in the circuit which opposes the growth of current in it. Therefore a certain amount of work has to be done by the current in

increasing the current from zero to a maximum value against the induced back e.m.f.. The work done by the current is stored in the magnetic field of the coil as potential energy. When the circuit is switched off, an e.m.f. is induced in the opposite direction which opposes the decay of current. Thus the work done by the current during the growth is recovered.

Let I be the current in the inductor L at any instant t.

Let the rate of growth of current be $\frac{dI}{dt}$

The back e.m.f. induced in the inductor $\varepsilon = -L\frac{dI}{dt}$

The work done in moving charge dq against this e.m.f is

$$dW = -\varepsilon dq = L\frac{dI}{dt}dq = L\frac{dq}{dt}dI = LI\,dI$$

Total work done in increasing the current from zero maximum value/I_0 is

$$W = \int_0^{I_0} LI\,dI = \frac{1}{2}L\,I_0^2$$

Thus the energy stored in the inductor is

$$U = \frac{1}{2}LI_0^2$$

Special case If $I_0 = 1$, then L = 2W

Thus, the coefficient of self-inductance is numerically equal to twice the work done in establishing magnetic induction accompanying unit current in the circuit.

3.15 Determination of Self – Inductance by Rayleigh's Method

The necessary circuit is shown in Fig 3.26. the coil, whose self-inductance L is to be measured, and a standard low resistance r (about 0.01Ω) are connected in the fourth arm of the Whetstone's bridge. A plug key K_3 is connected across r so that it may be short circuited. P, Q and R are non-inductive resistances.

(a) Initially , K_3 is kept closed. The ohmic resistance S of the inductance coil alone is included in the fourth arm. P is made equal to Q. then R is adjusted for no deflection in the B.G. by first pressing battery key K_1 and then galvanometer key K_2. Under this condition , no current flows through the galvanometer

(b) If now the galvanometer key K_2 is closed first and then the battery key K_1 then a throw θ_1 is observed in the galvanometer. This throw arises due to an extra emf $L\frac{di}{dt}$ induced in the coil while the current is growing.

If G is galvanometer resistance, then current through it due to induced e.m.f., is

$$i' = \frac{kL}{G}\frac{di}{dt}$$

Here , K is a constant which depends upon the relative resistance in the circuit. Hence the total charge passing through the galvanometer, as the current in the coil grows from zero to a steady maximum value i_o is given by

$$q = \int_0^{i_0} i'dt = \frac{kL}{G}\int_0^{i_0}\frac{di}{dt}dt = \frac{kL}{G}i_0 \text{............}(1)$$

If θ_1 be the first throw of the gal vanometer, then

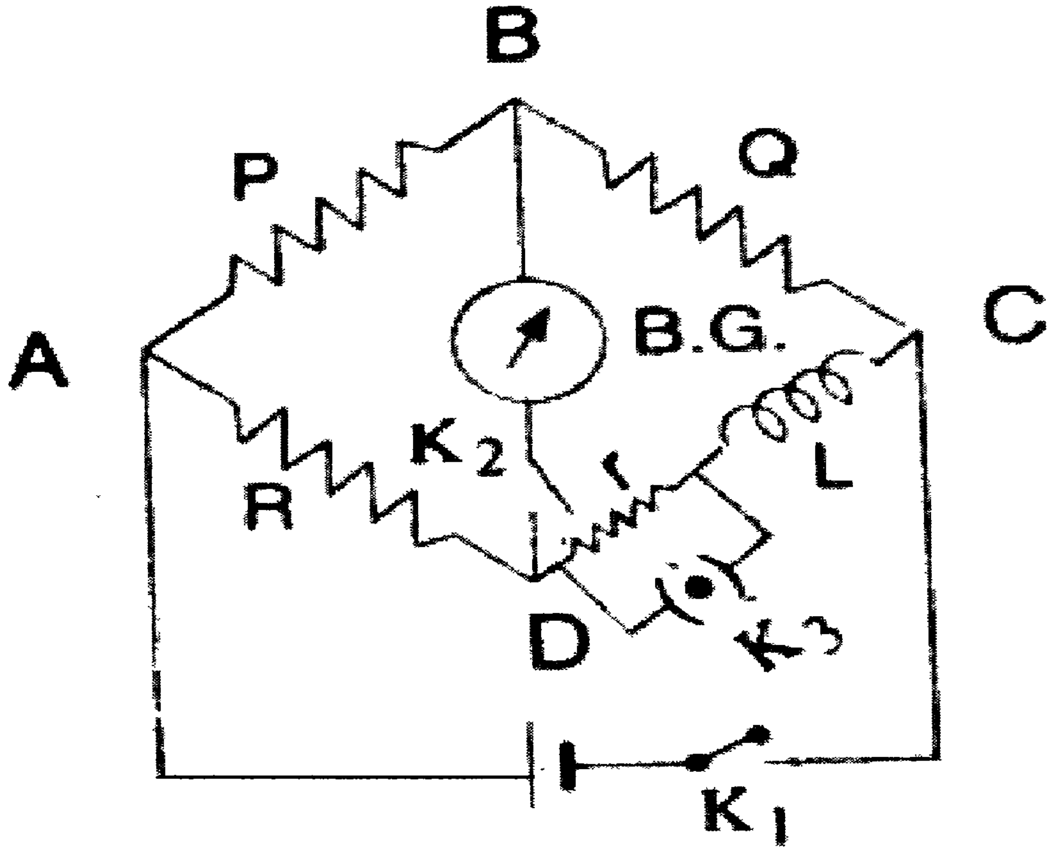


Fig 3.26

$$q = K\theta_1\left(1 + \frac{\lambda}{2}\right) \text{.........}(2)$$

$$\frac{kL}{G}i_0 = K\theta_1\left(1 + \frac{\lambda}{2}\right)$$ From Eqn.(1) and (2)

$$\frac{kL}{G}i_0 = \frac{T}{2\pi}\frac{c}{nBA}.\theta_1\left(1 + \frac{\lambda}{2}\right) \text{..................}(3)$$

(C) To eliminate k and I_0 the key K_3 is opened and the resistance r is included in the arm CD. As r is small, it does not affect the current i_0 in the current i_0 in the arm CD appreciably. But it will introduce an additional emf ri_0 in the arm CD. This causes a steady current $(kr/G)i_0$ through the galvanometer. K_1 is closed first and then K_2. The steady deflection φ in the galvanometer is noted. Then ,

$$\left(\frac{kr}{\ }\right)i_0 = \frac{c}{nBA}\varphi \text{...............}(4)$$

Here $\frac{c}{nBA}$ is the current reduction factor of the galvanometer.

Dividing Eqn.(3) by (4)

$$\frac{L}{r} = \frac{T}{2\pi}\frac{\theta_1}{\varphi}\left(1 + \frac{\lambda}{2}\right)$$

$$or\ L = \frac{rT}{2\pi}\frac{\theta_1}{\varphi}\left(1 + \frac{\lambda}{2}\right)$$

3.16 Mutual inductance – Determination using B.G

Consider a long air –cored solenoid with primary PP and secondary SS as shown in (Fig 3.28)

Let

N_1 = number of turns in the primary

N_2=number of turns in the secondary

A= Area of cross section.

l= length of the primary

I = Current in the primary

Magnetic field at any point inside the primary = $B = \frac{\mu_0 N_1 I}{l}$

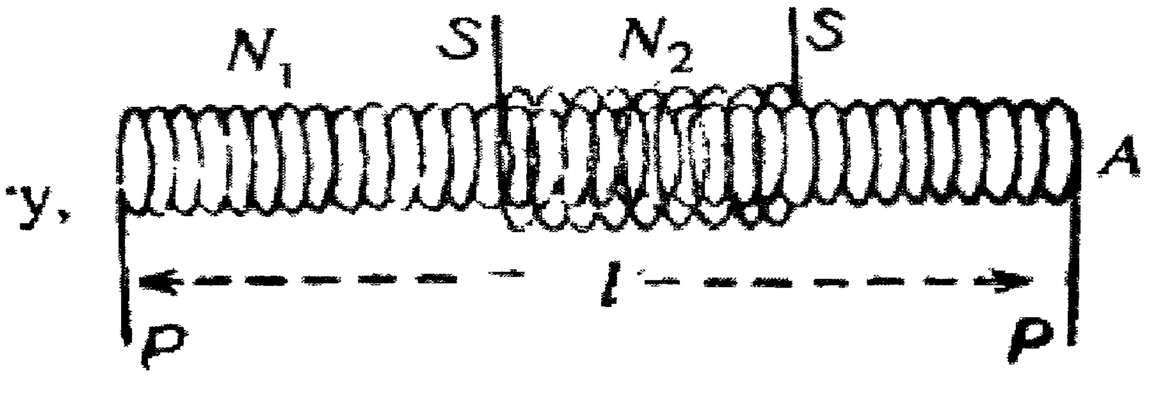


Fig g3.28

Magnetic flux through each turn of the primary is

$$BA = \frac{\mu_0 N_1 I A}{l}$$

Since the secondary is wound closely over the central portion of the primary, the same flux is also linked with each turn of the secondary.

Magnetic flux through each turn of the secondary= $\frac{\mu_0 N_1 I A}{l}$

Total magnetic flux through N_2 turns of the secondary=$\varphi = \frac{\mu_0 N_1 I A N_2}{l}$

By definition of mutual inductance,

$$M = \frac{\varphi}{I} = \frac{\mu_0 N_1 N_2 A}{l}$$

If the core is a material of permeability μ_r , then

$$M = \frac{\mu_0 \mu_r N_1 N_2 A}{l}\ henry$$

If there are a number of cores of area of cross-section A_1, A_2, A_3 etc., and relative perm abilities $\mu_{r1}, \mu_{r2}, \mu_{r3}$, etc

$$M = \frac{\mu_0 N_1}{l}[\mu_{r1} A_1 + \mu_{r2} A_2 + \mu_{r3} A_3 + \cdots] henry$$

Experimental Determination of Mutual Inductance:

Fig 3.29 represents the circuit arrangement for the measurement of mutual inductance between two coils P and S . C is a four-segment commutator and r is a very small resistance of the order of 0.01 ohm.

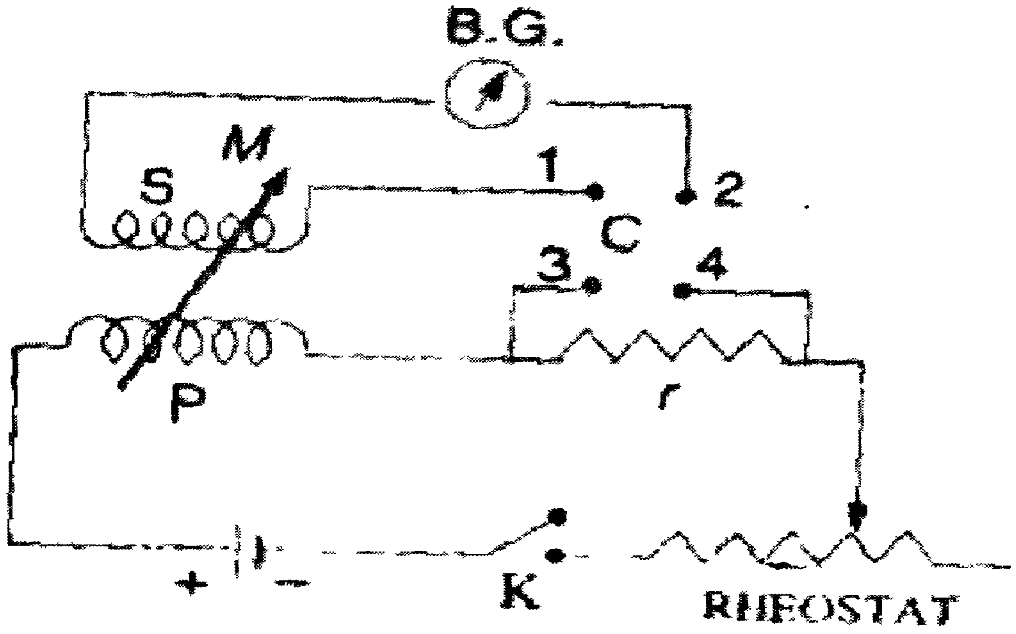


Fig 3.29

At first, 1 and 2 are connected together so that the secondary circuit is closed through the ballistic galvanometer (B.G). Now segments 3 and 4 are also connected together to short circuit the resistance r.

When the key K is pressed, the B.G. gives a throw. On pressing he key K the current in the primary slowly grows. Hence an induced emf is produced in the secondary. Let I be the instantaneous current in the primary.

The emf induced in the secondary = $\varepsilon = -M\frac{dI}{dt}$

The instantaneous current T in the secondary is

$$T = \frac{\varepsilon}{R} = \frac{M}{R}\frac{dI}{dt}\ (numerically)$$

Here, R is the total resistance of the secondary circuit.

Hence the total charge passing through the B.G as the current in the primary grows from zero to a steady maximum value I_0 in time internal t, is

$$q = \int_0^t I' dt = \int_0^t \frac{M}{R}.\frac{dI}{dt} dt = \int_0^{I_0} \frac{M}{R} dI = \frac{MI_0}{R} \text{.................}(1)$$

If θ_1 is the first throw in the B.G. due to this charge, then

$$q = \frac{T}{2\pi}.\frac{C}{NBA}.\theta_1\left(1+\frac{\lambda}{2}\right)$$

$$\frac{M}{R}I_0 = \frac{T}{2\pi}\frac{C}{NBA}.\theta_1\left(1+\frac{\lambda}{2}\right)...............(2)$$

To eliminate I_0 and C/(NBA) from Eq.(2), the contact between 1 and 2, and that between 3 and 4 are broken. The contact between 1 and 3, and that between 2 and 4 are made. The resistance r is now included in the primary circuit. As the value of r is very small, the steady current I_0 in the primary circuit is not altered. The potential difference across r is I_0r. It sends a steady current I_0r/R through the B.G. If φ be the steady deflection corresponding to this current, then

$$\frac{I_0 r}{R} = \frac{C}{NBA}.\varphi.........................(3)$$

Dividing Eq.(2) by Eq.(3), we get

$$M = \frac{rT}{2\pi}\frac{\theta_1}{\varphi}\left(1+\frac{\lambda}{2}\right)$$

Knowing the time period T and the logarithmic decrement λ of the ballistic galvanometer, M can be calculated.

3.17 Coefficient of coupling & Eddy currents

3.17.1 Coefficient of Coupling

Consider two coils having self-inductance L_1 and L_2 and number of turns N_1 and N_2 (Fig 3.30). L_1 and L_2 are the currents flowing through the two coils

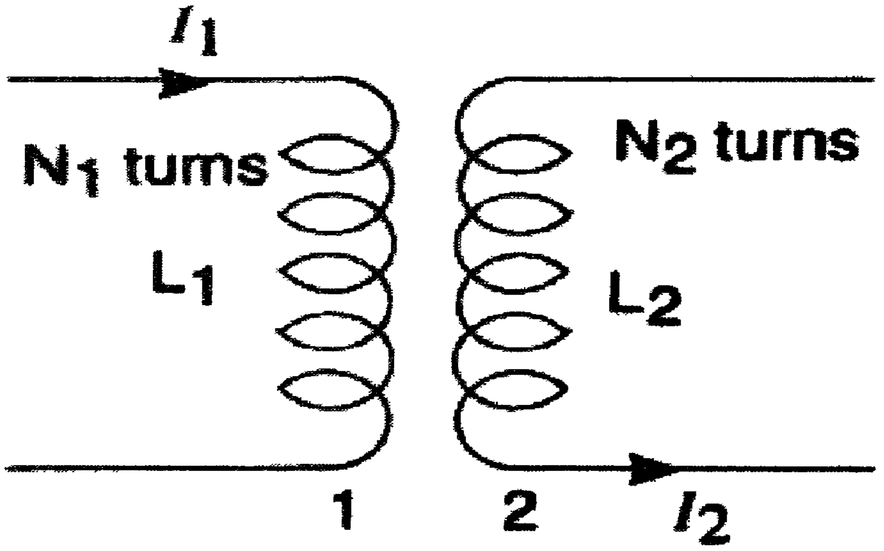


Fig 3.30

Let $\varphi_1 and\ \varphi_2$ be the magnetic fluxes linked with each turn of coils 1 and 2 due to their own currents I_1 and I_2 respectively

The self-inductance of the coils is given by

$$L_1 = \frac{N_1\varphi_1}{I_1}...................(1)$$

And

$$L_2 = \frac{N_2\varphi_2}{I_2}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Let φ_{12} be the flux per turn in the coil 1 due to current I_2 in coil 2. Similarly, φ_{21} is flux per turn linked with coil 2 due to current I_1 in coil 1.
Then the mutual inductance between them is given by

$$M = \frac{N_1\varphi_{12}}{I_2} = \frac{N_2\varphi_{21}}{I_1}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

The whole of the flux from one coil is linked with the other coil.

$$\varphi_{12} = \varphi_2$$

and

$$\varphi_{21} = \varphi_1$$

$$M = \frac{N_1\varphi_2}{I_2} = \frac{N_2\varphi_1}{I_1} \qquad \text{from Eqn.(3)}$$

$$M^2 = \frac{N_1N_2\varphi_1\varphi_2}{I_1I_2}\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

From Eq.(1) and Eq. (2) $L_1L_2 = \frac{N_1N_2\varphi_1\varphi_2}{I_1I_2}$ ……………………… (5)

Hence $M^2 = L_1L_2$

or $\quad M = \sqrt{(L_1L_2)}$ ………………………………(6)

In practise, however, the condition that whole of the flux from one coil links with the other, is not satisfied. This ration $\frac{M}{\sqrt{(L_1L_2)}}$ is known as the coefficient of coupling between the coils. It is denoted by k. thus

$$K = \frac{M}{\sqrt{(L_1L_2)}}$$

K is a number between 0 and 1, depending upon the geometry of the coils and their relative positions.
If K = 1(maximum value), there is no leakage of flux i.e., all the flux produced in one coil is linked with the other and $\frac{M}{\sqrt{(L_1L_2)}}$. This is the maximum possible value of M between the coils of self. Inductances L_1 and L_2. If K = 0, there is no coupling between the two coils.

3.17.2 Eddy Currents

Consider the coil of wire wound on a metal core (Fig 3.31). When an as source is used to drive current through the coil, an oscillating flux is set up through the metal. Because the flux through the dotted path shown in Fig (a) keeps changing, an emf is induced around this path. This induced emf, similar to other emf induced throughout the core, causes circular currents within the coil. We therefore see that induced currents, called eddy currents, flow in metal objects subjected to a changing magnetic field.

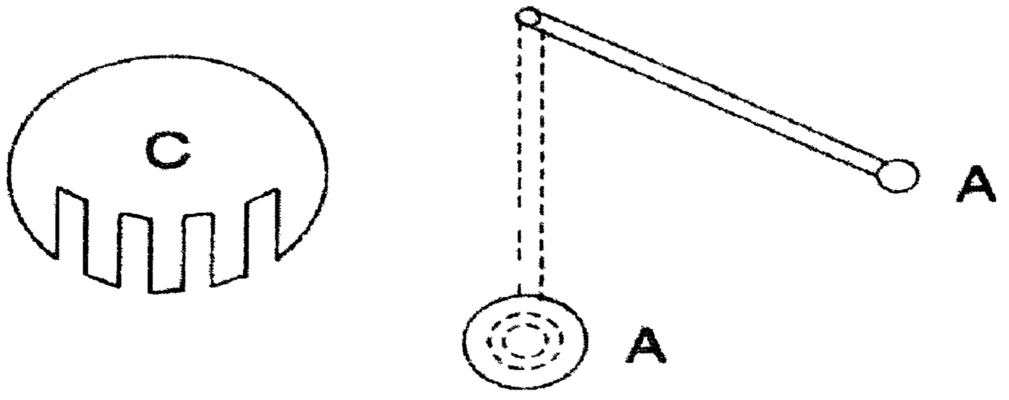


Fig 3.31

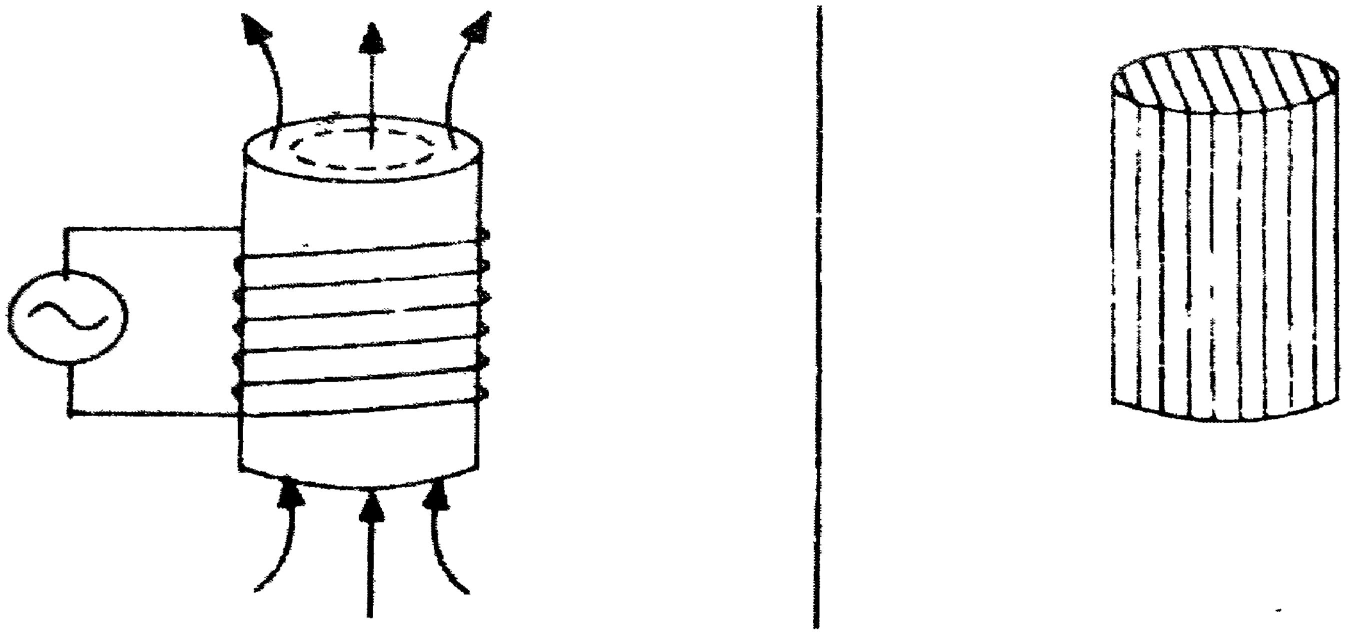


Fig 3.32

Eddy currents cause unwanted heating in transformers, motors and generators. To eliminated eddy currents, the metal cores of these devices are laminated. That is the core is split into thin slices insulated from each other (Fig 3.32). Because of the insulation barriers, current can no longer flow around paths such as the dotted circle shown in Fig (3.32). As a result unwanted heating of the metal is greatly reduced.

The production of eddy currents can be demonstrated using the apparatus shown in fig (3.32). A copper disc A is made to oscillate between the poles of an electromagnet. Initially, with the electromagnet switched off, the disc A is set into oscillations. The pendulum executes a large number of oscillations before stopping. Now if the electromagnet is switched on, the oscillating disc is brought to rest in a few oscillations. This is due to the eddy currents produced in the disc. The energy required for their production is taken from the oscillation. If the disc A is replaced by a slotted disc, C the eddy currents are reduced. Hence the disc C takes a longer time to come to rest when the magnetic field is switched on.

3.17.3 Uses of Eddy Currents

(1) In aperiodic galvanometers,the coil is wound on a metal frame. When oscillating between the pole pieces of the magnet, the eddy currents induced in the frame damp these oscillations. Hence the coil comes to rest quickly.

(2) If a rotating magnetic field is applied to a lightly mounted metal disc, the eddy currents set up in the disc make the disc rotate in the direction which tends to prevent the change in flux. This is the principle of the induction motor.

(3) The induction furnace uses eddy currents to produce heat in metallic objects. In recent years induction heating has been used in industry to heat metals. The specimen is placed in a magnetic field of frequency about a few mega hertz. This process has also been used to prepare some of the alloys by melting the constituents in vacuum.

(4) Eddy currents can be used to produce a braking effect in moving vehicles. A metallic drum is attached to the axle of the moving train. When a magnetic field is suddenly applied to the rotating drum, eddy currents are produced. This tends to oppose the rotation and leads to a braking effect.

3.18 Summary

(Let us Sum up)

- **Maxwell's cork screw Rule:** If a right handed screw is turned to advance along the conductor in the direction of the current, the direction of rotation of the screw gives the direction of the lines of force.
- **Right hand clasp Rule:** Clasp the conductor in the right hand with the thumb pointing in the direction of the current. Then the direction of bend of the rest of the fingers gives the direction of the magnetic lines of force.
- The magnitude of the magnetic induction at a point is the ratio of the force on a moving charge at that point and the product of the charge and component of its velocity along a direction normal to the induction.
- **Biot-Savart's Law:**

$$dB = \left(\frac{\mu_0}{4\pi}\right)\frac{i\,(dl\sin\theta)}{r^2}$$

- Magnetic Field at the centre of a Circular Coil Carrying Current:

$$B = \frac{n\mu_0 i}{2r}$$

- **Ampere's Law:** The line integral $\oint B.dl$ for a closed curve is equal to $\mu_0$times the net current i through the area bounded by the curve. That is,$\oint B.dl = \mu_0 i$

- **Principle of Moving Coil Ballistic Galvanometer:** When a current is passed though a coil, suspended freely in a magnetic field, it experiences a force in a direction given by Fleming's left hand rule.
- $Ballistic\ reduction\ factor\ (K) = \left(\frac{T}{2\pi}\right)\left(\frac{C}{NBA}\right)$

3.19 Unit - end exercises

1. Define Ampere's Swimming Rule
2. Define magnetic induction
3. Define Magnetic flux
4. State and Explain Biot and Savart's law
5. Obtain an expression for the field at the centre of a current carrying circular coil.
6. Derive an expression for magnetic field inside a solenoid carrying current
7. Derive an expression for the force on a current carrying element placed in a uniform magnetic field
8. Derive an expression for the torque acting on a current carrying loop in a magnetic field.
9. Explain damping correction for a ballistic galvanometers
10. Described the damping correction in a B.G
11. Distinguish between ballistic and an aperiodic galvanometer
12. What are eddy currents? Mention any two of its applications.
13. Describe the Rayleigh's method of determining the self inductance of a coil with theory.
14. Give the theory of moving coil Ballistic galvanometer. How is the damping correction made?
15. Obtain the theory of a moving coil Ballistic galvanometer with damping correction,

3.20 Problems for discussions

1. A circular coil has a radius of 0.1m and a number of turns of 50. Calculate the magnetic induction at a point (i) on the axis of the coil and distance 0.2 m from the centre; (ii) at the centre of the coil, when a current of 0.1 A flows in it.
2. In the Bohr model of the hydrogen atom, the electron circulates around the nucleus in a path of radius 5.29×10^{-11} m at a frequency of 6.58×10^{15} Hz. Find the magnitude of the magnetic induction at the centre of the orbit. What is its dipole moment?

3. The current sensitivity of a ballistic galvanometer is 2.2 x 10^{-9} amperes for a deflection of 1 mm on a scale kept at a distance of 1 metre. Calculate the charge sensitivity of the galvanometer if time period of the coil is 6.2 seconds.
4. A solenoid having an air core and 10 cm long has 100 turns and its area of cross-section is 5sq.cm. Find the co-efficient of self-inductance of the solenoid.
5. Calculate the self-inductance of a solenoid having 1000 turns and length 1 m. The area of cross-section is $7cm^2$ and the relative permeability of the core is 1000.
6. In the Bohr model of the hydrogen atom, the electron circulates around the nucleus in a path of radius 5.29 x 10^{-11} m at a frequency of 6.58 x 10^{15} Hz. Find the magnetic induction at the centre of the orbit. What is its dipole moment?

3.21 Answer to check your progress & Problems for discussions

Answer to check your progress:

1. Ans:

$$B = \frac{2\sqrt{2}\mu_0 i}{\pi d} = \frac{2\sqrt{2}\,X\,(4\pi\,X\,10^{-7})X\,1}{\pi\,X\,1} = 8\sqrt{2}\,X\,10^{-7}\,Wb\,m^{-2}.$$

2. Ans:

The pair F and v, and F and B are always at right angles. Vectors V and B may have any angles between them.

3. Ans:

Both are inverse square laws. In Coulomb's law electrical force acts along r on stationary charge. In Biot-Savart law, magnetic force acts perpendicular to r.

4. Ans:

$$\frac{T}{2\pi}\left(\frac{\text{Charge Sensitivity}}{\text{Current sensitivity}} = \frac{T}{2\pi}\right)$$

5. Ans :

Dead – beat Galvanometer	Ballistic Galvanometer
1. It measure steady current 2. The steady deflection measures the current.	1. It measures charge 2. The throw measures the charge 3. The coil is wound on a

3. The coil is wound on a metal frame to increase electromagnetic damping 4. The coil is non-oscillatory due to large damping 5. The coil rotates due to constant torque.	non-metallic frame to reduce electromagnetic damping 4. The coil is oscillatory due to small damping 5. The momentary passage of charge causes impulse on the coil. The torque is zero when the coil rotates.

6. Ans :

As described by the magnetic moment μ is given by

$$\mu = NIA = (10)(3.0\,A)\pi\,(0.050m)^2$$
$$= 0.24\,Am^2$$

The magnitude of the torque needed to hold the new orientation is given by

$$\tau = \mu\,B\sin\theta = (0.24\,Am^2)(0.010T)(\sin 90^0)$$
$$= 2.4\,X\,10^{-2}Nm.$$

Answers to Problems for discussions:

***1.* Solution.**

i) Here , a = 0.1m , N=50, x =0.2 and i = 0.1 A.B = ?

$$B = \frac{\mu_0 N i a^2}{2(a^2+x^2)^{\frac{3}{2}}} = \frac{(4\pi\,X\,10^{-7})X\,50\,X\,0.1\,X\,(0.1.)^2}{2[(0.1)^2 + (0.2)^2]^{\frac{3}{2}}} = 2.81X10^{-6}T$$

ii) At the centre.

$$B = \frac{\mu_0 N i}{2a} = \frac{(4\pi\,X\,10^{-7})X\,50\,X\,0.1}{2\,X\,0.1} = 3.14\,X\,10^{-5}T$$

2. **Solution**. Current = charge / time = eν

Where e is the electronic and ν is the frequency of revolution.
i = eν = (1.602 x 10^{-19}) (6.58 x 10^{15}) = 1.054 x 10^{-3} A

Magnetic induction at the centre of the orbit is

$$B = \frac{\mu_0 i}{2a} = \frac{(4\pi \text{ x } 10^{-7}) \text{ x } (1.054 \text{ x } 10^{-3})}{2 \text{ x } (5.29 \text{ x } 10^{-11})} = 12.52T$$

let$A = \pi a^2$be the area of the current loop. Then

M = Ai is called the magnetic dipole moment of the current loop.

$M = Ai = \pi a^2 i = \pi(5.29 \times 10^{-11})^2(1.50 \times 10^{-3})$

$= 9.266\ 10^{-24}\ Am^2$

3. **Solution** Charge sensitivity $= \frac{T}{2\pi} \times currnet\ sensitivity$

$$= \frac{6.2}{2 X 3.14} \times (2.2 X 10^{-9})$$

$$= 2.17 \times 10^{-9} \frac{Coulombs}{mm}.$$

4. **Solution.** Here l = 10cm = 0.2 m. N = 100, A = 5 sq.cm = 5 X 10^{-4} m^2

$$L = \frac{\mu_0 N^2 A}{l}$$

$$\frac{(4\pi \times 10^{-7})(100 \times 100)(5 \times 10^{-4})}{0.1} = 62.8 \times 10^{-6}\ henry$$

5. **Solution.** Here l = 1m, N=1000, A = 7 X $10^{-4}m^2$, μ_r= 1000

$$L = \frac{\mu_0 \mu_r N^2 A}{l} = \frac{1000 X (4\pi \times 10^{-7})(1000)^2(7 \times 10^{-4})}{1} = 0.88\ henry$$

6. **Solution.** Current = Charge / time = ev

Where e is the electronic charge and v is the frequency of revolution.

$i = ev = (1.602 \times 10^{-19})(6.58 \times 10^{15}) = 1.054 \times 10^{-3}$ A

magnetic induction at the centre of the orbit is

$$B = \frac{\mu_0 i}{2a} = \frac{(4\pi \times 10^{-7}) \times (1.054 \times 10^{-3})}{2 \times (5.29 \times 10^{-11})} = 12.52\ T$$

Let A $= \pi a^2$ *be the area of the current loop. then*

M = Ai is called the magnetic dipole moment of the current loop

$M = Ai = \pi a^2 i = \pi\ (5.29 \times 10^{-11})^2(1.054 \times 10^{-3})$

$=9.266\ 10^{-24}\ Am^2$.

3.22 Suggested Readings

1. Electricity and Magnetism - S Mahajan And A A Rangwala ,Tata McGrew Hill
2. Electricity and Magnetism - Dr.K.K.Tewari S.Chand & Co,2002.
3. Electricity and Magnetism with Electronics -D.N.Vasudeva S.Chand & Co,2002.
4. Electricity and Magnetism - Narayanamoorthy, Nagarathinam. 2nd Revised Edition

UNIT IV: LCR circuits

Structure

4.1 Introduction
4.2 Objectives
4.3 Growth and decay of current in LR circuit
4.3.1. Growth of current in a circuit containing L and R
4.3.2. Decay of current in a circuit containing L and R
4.4 Growth and decay of charge in CR circuit
4.4.1. Growth of Charge
4.4.2. Decay of charge: (Discharging of C through R)
4.5 Growth and decay of charge in a circuit with L,C and R in series
4.5.1. Growth of charge in a circuit with L, C and R in series
4.5.2. Discharge of a capacitor through a L and R in series (Decay of charge in LCR circuit
4.6 Determination of High Resistance by Leakage (B.G)
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4.1 Introduction

Transient phenomena are phenomena that exist only for a short while and are not simple periodic functions of time. The different parts of the circuit invariably have some resistance, capacitance and inductance associated with them. When the switch is put on, the current starts flowing steadily, there lies a transient phase in which the current is building up. The resistance retards the flow of current, the capacitances take their own time in charging up to their equilibrium potentials and inductances act like inertia and develop back emf's which retard the forward flow of current. Thus depending upon the values of R, L and C in the circuit, there will be a transient phase of some duration during which the current will build up.

Based on the phenomenon of electromagnetic induction, a dynamo is used to generate alternating currents. Practically all transmission of electrical power is now achieved using alternating currents. The theory of Wheatstone bridge using dc currents and use of Kirchhoff's laws for analyzing dc circuits are extended to ac bridges.

4.2 Objectives

After studying this unit, you will be able to

- compute the growth and decay of current in LR, CR and LCR series circuit.
- determine the high resistance by leakage
- define mean and RMS value of alternating current
- know about the effect of AC applied to LR and CR circuit
- explain the theory of series and parallel resonance circuit
- explain power in an AC circuits
- define wattles current and Q – factor
- explain choke and skin effect and
- describe the AC bridges

4.3 Growth and decay of current in LR circuit

4.3.1 Growth of current in a circuit containing L and R.

Consider a circuit having an inductance L and a resistance R connected in series to cell of steady emf E (Fig.4.1). When the key K is pressed, there is a gradual growth of current in the circuit from zero to maximum value Io. Let I be the instantaneous current at any instant.

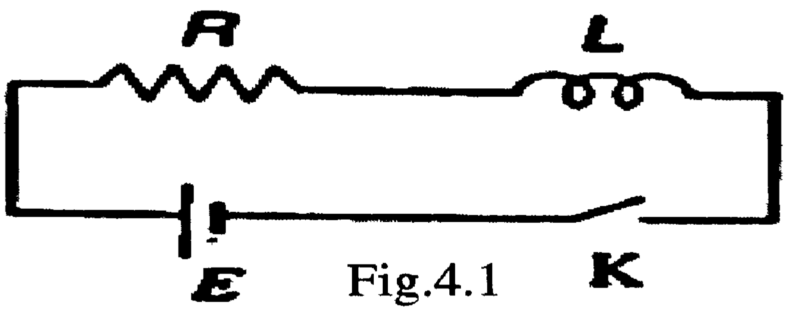


Fig.4.1

Then, the induced, back emf $\varepsilon = -L\frac{dI}{dT}$

$$E = RI + L\frac{dI}{dT} \qquad (1)$$

When the current reaches the maximum value Io, the back emf,

$$L\frac{dI}{dT} = 0$$

Hence, $E = R\,I_o$ (2)

Substituting this value for E in Eq. (1),

$$RI_o = RI + L\frac{dI}{dT} \text{ or } R(I_o - I) = L\frac{dI}{dT}$$

$$\text{or } \frac{dI}{I_o - I} = \frac{R}{L}dt$$

Integrating,

$$-\log(I_o - I) = \frac{R}{L}t + C \qquad (3)$$

Here C is the constant of integration.

When t = 0, I = 0; $-\log_e I_o = C$

Substituting this value of C in Eq. (3),

$$-\log(I_o - I) = \frac{R}{L}t - \log_e I_o \text{ (or) } -\log_e(I_o - I) - \log_e I_o = -\frac{R}{L}t$$

$$-\log_e \frac{(I_o - I)}{I_o} = -\frac{R}{L}t$$

$$\frac{(I_o - I)}{I_o} = e^{-\left(\frac{R}{L}\right)t} \text{ or } 1 - \frac{I}{I_o} = e^{-\left(\frac{R}{L}\right)t}$$

$$I = I_o\left(1 - e^{-\left(\frac{R}{L}\right)t}\right) \qquad (4)$$

Eq. (4) gives the value of the instantaneous current in the LR circuit. The quantity (L/R) is called the time constant of the circuit. If

$$\frac{L}{R} = t,\ I = I_o(1 - e^{-1}) = I_o(1 - \frac{1}{e}) = 0.632 I_o.$$

Thus, the time constant L/R of an L-R circuit is the time taken by the current to grow from zero to 0.632 times the steady maximum value of current in the circuit.

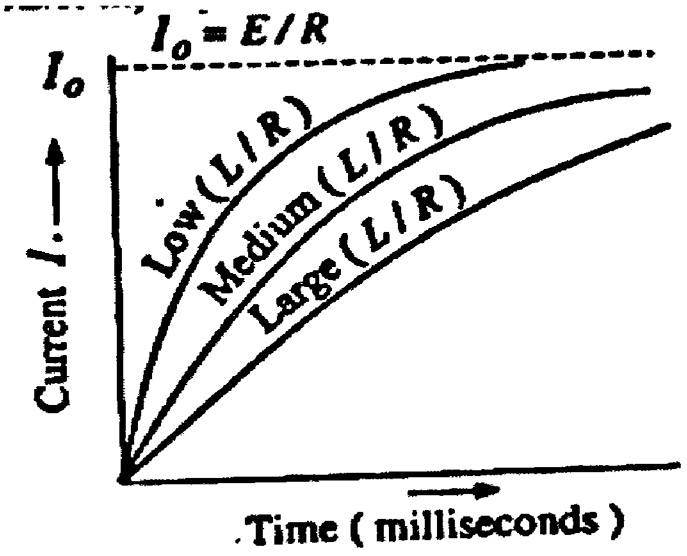


Fig.4.2

Similarly, when t = 2L/R, 3L/R...... the value of current will be 0.8647, 0.9502..., of the final maximum current.

When t = 0, I = 0 and

When $t \to \infty$ $I = I_o$

Greater the value of L/R, longer is the time taken by the current I to reach its maximum value (Fig.4.2)

$$I_o = E / R$$

4.3.2 Decay of current in a circuit containing L and R

When the circuit is broken, an induced emf equal to $-L\frac{dI}{dT}$ is again produced in the inductance L and it slows down the rate of decay of the current. The current in the circuit decays from the maximum value Io to zero. During the decay, let I be the current at time t. In this case E = 0.

The emf equation for the decay of current is

$$0 = RI + L\frac{dI}{dT} \qquad (1)$$

$$\frac{dI}{I} = -\frac{R}{L}dt$$

Integrating $\log_e I = -\frac{R}{L}t + C$ Where C is a constant

When t = 0, $I = I_o$; $\log_e I_o = C$

$$\log_e I = -\frac{R}{L}t + \log_e I_o \text{ or } \log_e\left(\frac{I}{I_o}\right) = -\frac{R}{L}t$$

$$\frac{I}{I_o} = e^{-\frac{R}{L}t}$$

$$I = I_o e^{-\left(\frac{R}{L}\right)t} \tag{2}$$

Eq. (2) represents the current at any instant t during decay.

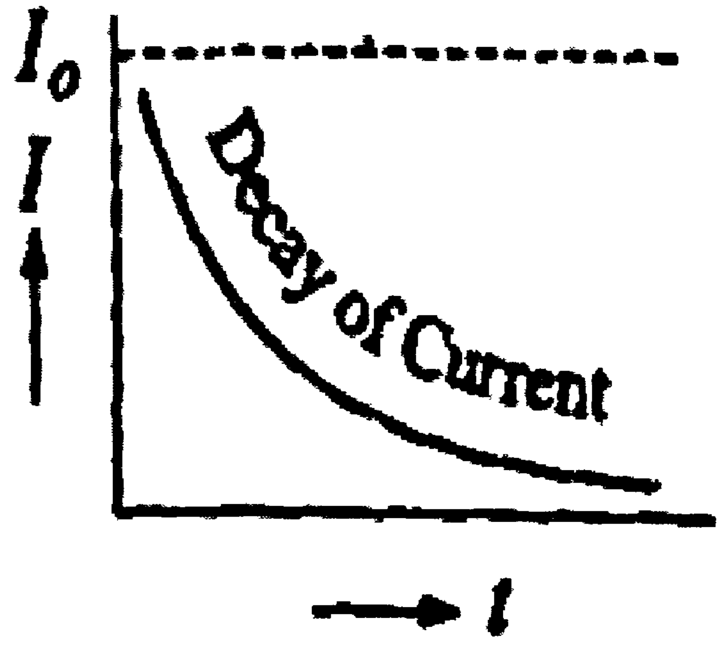


Fig.4.3

A graph between current and time is shown in Fig.4.3.

When $\frac{L}{R} = t,\ I = I_o e^{-1} = I_o \frac{1}{e} = 0.365 I_o$

$\frac{2L}{R} = t,\ I = I_o e^{-2} = 0.1035 I_o$

$\frac{3L}{R} = t,\ I = I_o e^{-3} = 0.05 I_o$

Therefore, the time constant L/R of a R-L circuit may be defined as the time in which the current in the circuit falls to (1/e) of its maximum value when external source of emf is removed.

The rate of decay of current is

$$\frac{dI}{dT} = -\frac{R}{L} I_{oe}^{\left(-\frac{R}{L}\right)t} = -\frac{R}{L} I$$

Thus it is clear that greater the ratio R/L, or smaller the time constant L/R, the more rapidly does he current die away (Fig.4.4)

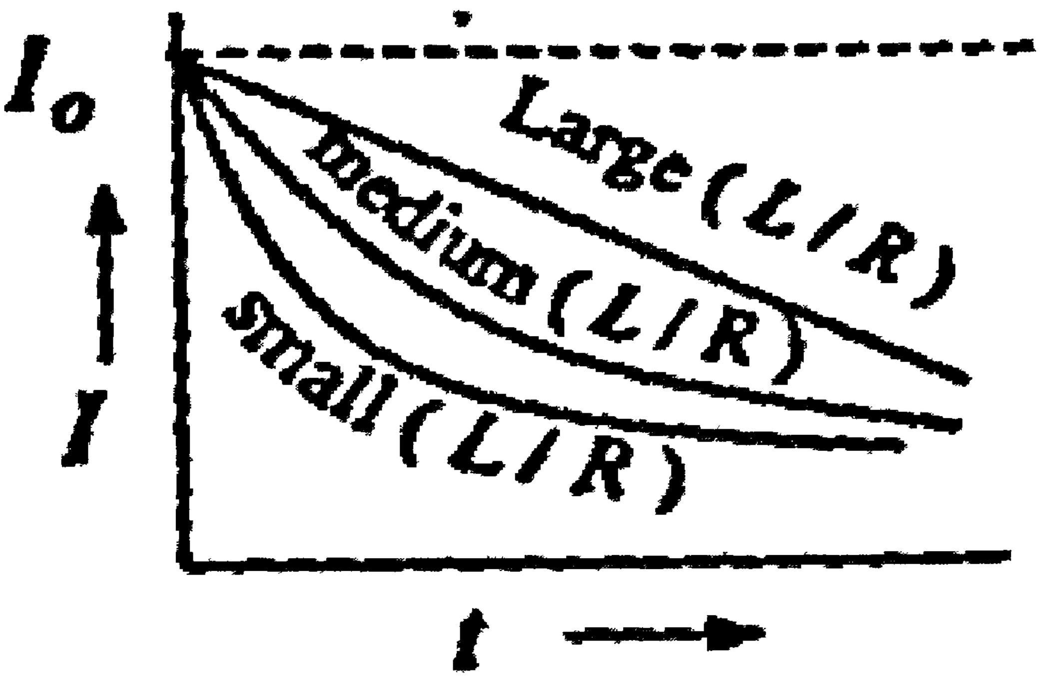


Fig.4.4

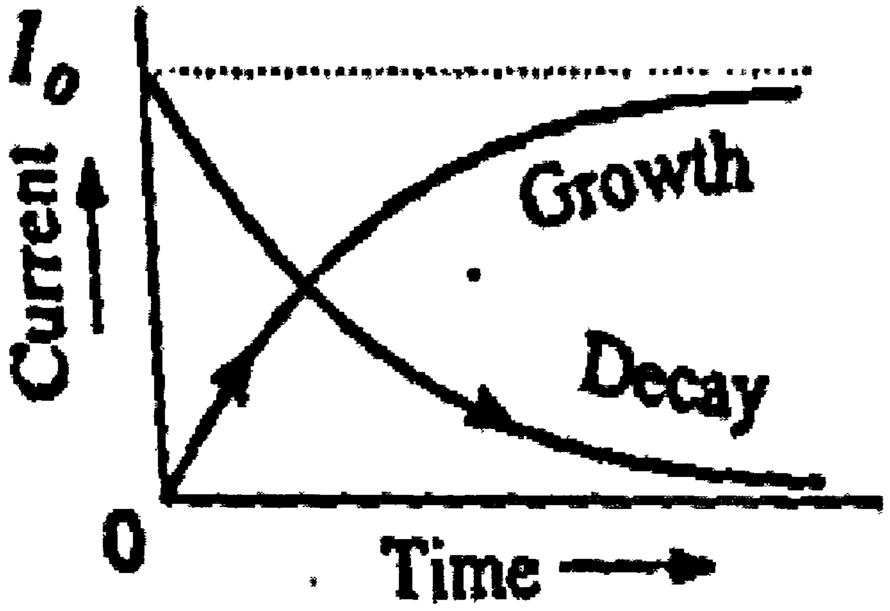


Fig.4.5

Fig.4.5 shows that the growth and decay curves are complementary.

Check your progress
1. Define the time constant L/R of a L-R circuit. 2. Write the equations for decay of current in a circuit containing L and R. 3. With reason state which of the following sets is suitable for obtaining rapid growth and decay of current in L-R circuit: I set: L = 1 mH, R = 1000 Ω; II set: L = 10 mH, R = 10 Ω. Ans: --- ---

4.4 Growth and decay of charge in CR circuit

4.4.1. Growth of Charge

A capacitor C and a resistance R are connected to a cell of emf E through a Morse key K (Fig. 4.6). When the key is pressed, a momentary current I flows through R. At any instant t, let Q be the charge on the capacitor of capacitance C

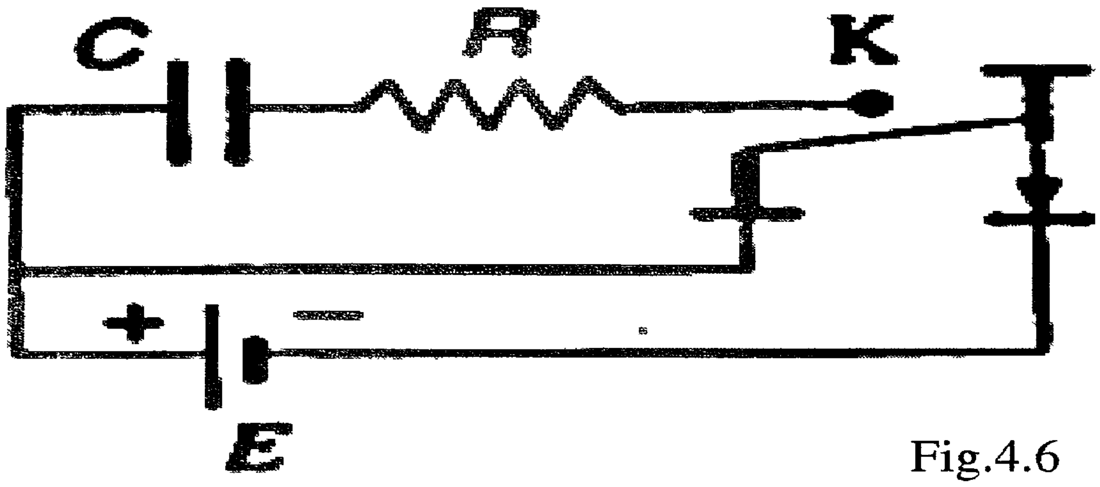


Fig.4.6

P.D. across capacitor = Q/C

P.D. across resistor = RI

The emf equation of the circuit is

$E = (Q/C) + RI$ (1)

$E = (Q/C) + R\,(dQ/dt)$ $I = dQ/dt$

The capacitor continues getting charged till it attains the maximum charge Q_0. At that instant $I = dQ/dt = 0$.

The P.D. across the capacitor is $E = Q_0/C$.

i.e., when $Q = Q_0$, $\frac{dQ}{dt} = 0$ and $E = \frac{Q_0}{C}$

$$\therefore \frac{Q_0}{C} = \frac{Q}{C} + R\frac{dQ}{dt}$$

$$(Q_0 - Q) = CR\frac{dQ}{dt}$$

$$\left(\frac{dQ}{Q_0 - Q}\right) = \frac{dt}{CR}$$

Integrating, $-\log_e (Q_0 - Q) = \frac{t}{CR} + K$

Where K is a constant.

When $t = 0$, $Q = 0$ $\therefore -\log_e Q_0 = k$

$$-\log_e(Q_0 - Q) = \frac{t}{CR} - \log_e Q_0$$

$$\log_e(Q_0 - Q) = -\frac{t}{CR} + \log_e Q_0$$

$$\log_e(Q_0 - Q) - \log_e Q_0 = -\frac{t}{CR}$$

$$\log_e\left(\frac{Q_0 - Q}{Q_0}\right) = -\frac{t}{CR}$$

$$\left(\frac{Q_0 - Q}{Q_0}\right) = e^{-\left(\frac{t}{CR}\right)} \quad \text{or } 1 - \frac{Q}{Q_0} = e^{-\left(\frac{t}{CR}\right)}$$

$Q = Q_0 (1 - e^{-(\frac{t}{CR})})$

The term CR is called time constant of the circuit.

At the end of time t = CR, $Q = Q_0 (1 - e^{-1}) = 0.632\ Q_0$.

Thus, the time constant may be defined as the time taken by the capacitor to get charged to 0.632 times its maximum value.

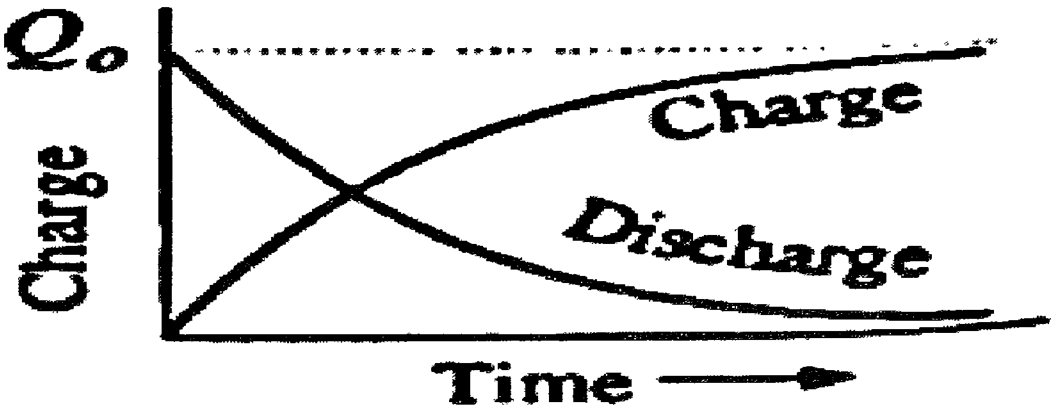


Fig.4.7

The growth of charge is shown in Fig. 4.7.

$$\frac{dQ}{dt} = \frac{Q_0}{CR} e^{-\frac{t}{CR}} = \frac{1}{CR}(Q_0 - Q)$$

Thus it is seen that smaller the product CR, the more rapidly does the charge grow on the capacitor.

The rate of growth of the charge is rapid in the beginning and it becomes less and less as the charge approaches nearer and nearer the steady value.

4.4.2. Decay of charge: (Discharging of C through R)

Let the capacitor having charge Q_0 be now discharged by releasing the Morse key K (Fig. 4.6). The charge flows out of the capacitor and this constitutes a current. In this case E = 0.

$$R\frac{dQ}{dt} + \frac{Q}{C} = 0 \qquad \text{.....................(1)}$$

Or $\frac{dQ}{Q} = -\frac{1}{CR} dt$

Integrating, $\log_e Q = -\frac{t}{CR} + K$, where K is constant

When t = 0, $Q = Q_0$; $\therefore \log_e Q_0 = k$

$\log_e Q = -\frac{t}{CR} + \log_e Q_0$

Or $\log_e \frac{Q}{Q_0} = -\frac{t}{CR}$ or $\frac{Q}{Q_0} e^{-t/CR}$

$$\therefore Q = Q_0 e^{-t/CR} \qquad \text{............(2)}$$

This shows that the charge in the capacitor decays exponentially and becomes zero after infinite interval of time. (Fig. 4.7).

The rate of discharge is

$$I = \frac{dQ}{dt} = -\frac{Q_0}{CR} e^{-(\frac{t}{CR})} = -\frac{Q}{CR} \qquad \text{..................(3)}$$

Thus. smaller the time constant CR, the quicker is the discharge of the capacitor.
In Eq. (2), if we put t = CR, then $Q = Q_0e^{-1} = 0.368\ Q_0$
Hence time constant may also be defined as the time taken by the current to fall from maximum to 0.368 of its maximum value.

Check your progress
4. Define time constant of a CR circuit.
Ans: --

4.5 Growth and decay of charge in a circuit with L, C and R in series

4.5.1. Growth of charge in a circuit with L, C and R in series

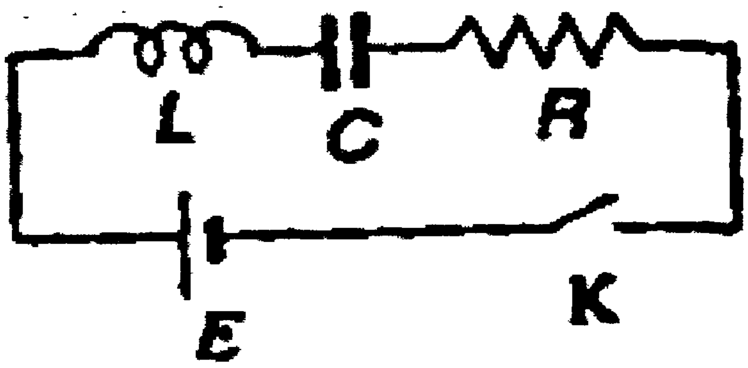


Fig 4.8

Consider a circuit containing an inductance L, capacitance C and resistance R joined in series to a cell of emf E (Fig. 4.8). When the key K is pressed. the capacitor is charged. Let Q be the charge on the capacitor and I the current in the circuit at an instant t during charging. Then, the p.d across the capacitor is Q / C and the self-induced emf in the inductance coil is L (dI/dt), both being opposite to the direction of E. The P.D. across the resistance R is RI.

The equation of emf's is

$$L\frac{dI}{dt} + RI + \frac{Q}{C} = E \quad \text{.........................(1)}$$

But $I = \frac{dQ}{dt}$ and $\frac{dI}{dt} = \frac{d^2Q}{dt^2}$

$$\therefore \frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = E$$

Or $$\frac{d^2Q}{dt^2} + \frac{R}{L}\frac{dQ}{dt} + \frac{(Q-CE)}{LC} = 0$$

Putting $\frac{R}{L} = 2b$ and $\frac{1}{LC} = K^2$, we have

$$\frac{d^2Q}{dt^2} + 2b\frac{dQ}{dt} + K^2(Q - CE) = 0 \quad \text{.......................(2)}$$

Let $x = (Q - CE)$ then $\frac{dx}{dt} = \frac{dQ}{dt}$ and $\frac{d^2x}{dt^2} = \frac{d^2Q}{dt^2}$

Eq. (2) becomes, $\frac{d^2x}{dt^2} + 2b\frac{dx}{dt} + k^2x = 0$.......................(3)

Hence the most general solution of Eq. (3) is

$$x = Ae^{\left[-b+\sqrt{(b^2-k^2)}\right]t} + Be^{\left[-b-\sqrt{(b^2-k^2)}\right]t}$$

Now, CE = Q_0 = final steady charge on the capacitor.

$\therefore$ x = Q – CE = Q – Q_0

Hence $Q - Q_0 = Ae^{\left[-b+\sqrt{(b^2-k^2)}\right]t} + Be^{\left[-b-\sqrt{(b^2-k^2)}\right]t}$

Or $$Q = Q_0 + Ae^{\left[-b+\sqrt{(b^2-k^2)}\right]t} + Be^{\left[-b-\sqrt{(b^2-k^2)}\right]t} \quad \ldots\ldots\ldots (4)$$

Using initial conditions:

At t = 0, Q = 0

$\therefore$ $0 = Q_0 + (A+B)$ or $A + B = -Q_0$ .... (5)

Or $$\frac{dQ}{dt} = A\left[-b+\sqrt{(b^2-k^2)}\right]e^{\left[-b+\sqrt{(b^2-k^2)}\right]t} + B\left[-b-\sqrt{(b^2-k^2)}\right]e^{\left[-b-\sqrt{(b^2-k^2)}\right]t}$$

At t = 0, $\frac{dQ}{dt} = 0$

$$0 = A\left[-b+\sqrt{(b^2-k^2)}\right] + B\left[-b-\sqrt{(b^2-k^2)}\right]$$

$\sqrt{(b^2-k^2)}\,[A - B] = b\,(A+B) = -bQ_0$

Or $A - B = -\frac{Q_0 b}{\sqrt{(b^2-k^2)}}$ .... (6)

Solving Eqs. (5) and (6),

$$A = -\frac{1}{2}Q_0\left[1 + \frac{b}{\sqrt{(b^2-k^2)}}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

$$B = -\frac{1}{2}Q_0\left[1 - \frac{b}{\sqrt{(b^2-k^2)}}\right] \quad \ldots\ldots\ldots\ldots (8)$$

Substituting the values of A and B in Eq.(4), we have

$$Q = Q_0 - \frac{1}{2}Q_0e^{-bt}\left[\left(1 + \frac{b}{\sqrt{(b^2-k^2)}}\right)e^{\sqrt{(b^2-k^2)}t} + \left(\left(1 - \frac{b}{\sqrt{(b^2-k^2)}}\right)e^{\sqrt{(b^2-k^2)}t}\right)\right] (9)$$

Case I: If $b^2 > k^2$, $\sqrt{(b^2-k^2)}$ is real. The charge on the capacitor grows exponentially with time and attains the maximum value Q_0 asymptotically (curve 1 of Fig. 4.9). The charge is known as over damped or dead beat.

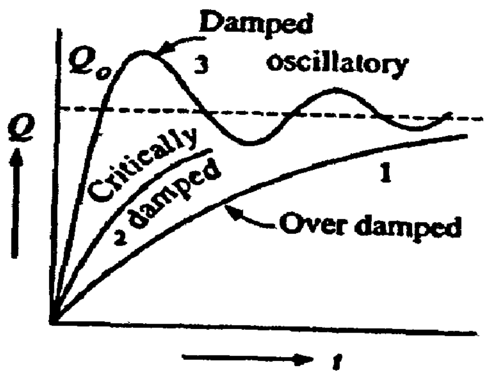


Fig.4.9

Case II: If $b^2 = k^2$ the charge rises to the maximum value Q_0 in a short time (curve 2 of Fig.4.9). Such a charge is called critically damped.

Case III: If $b^2 < k^2$, $\sqrt{(b^2-k^2)}$ is imaginary.

Let $\sqrt{(b^2-k^2)}$ = iω where i = $\sqrt{-1}$ and ω = $\sqrt{(b^2-k^2)}$

Eq.(9) may be written as

$$Q = Q_0 - \frac{1}{2}Q_0e^{-bt}\left[\left(1+\frac{b}{i\omega}\right)e^{i\omega t} + \left(1-\frac{b}{i\omega}\right)e^{-i\omega t}\right]$$

$$Q = Q_0 - Q_0e^{-bt}\left[\frac{e^{i\omega t}+e^{-i\omega t}}{2} + \frac{b}{\omega}\frac{e^{i\omega t}-e^{-i\omega t}}{2i}\right]$$

$$Q = Q_0 - Q_0e^{-bt}\left(cos\omega t + \frac{b}{\omega}sin\omega t\right)$$

$$Q = Q_0\left[1 - \frac{e^{-bt}}{\omega}(\omega cos\omega t + bsin\omega t)\right]$$

Let ω = k sin ∝ and b = k cos∝ so that tan ∝ = ω/b.

$$Q = Q_0\left[1 - \frac{ke^{-bt}}{\omega}(ksin \propto cos\omega t + kcos \propto sin\omega t)\right]$$

Or

$$Q = Q_0\left[1 - \frac{ke^{-bt}}{\omega}(\sin(\omega t{+}\propto))\right] \quad \text{..........} \quad (10)$$

$$Q = Q_0\left[1 - \frac{e^{-\frac{R}{2L}t}\sqrt{\frac{1}{LC}}}{\sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}}}.sin\left[\left(\sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}}\right)t{+}\propto\right]\right]$$

This equation represents a damped oscillatory charge as shown by the curve (3). The charge oscillates above and below Q_0 till it finally settles down to Q_0 value. The frequency of oscillation in the circuit is given by

$$v = \frac{\omega}{2\pi} = \frac{\sqrt{k^2-b^2}}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}}$$

When R = 0, $v = \frac{1}{2\pi\sqrt{LC}}$

4.5.2 Discharge of a capacitor through a L and R in series (Decay of charge in LCR circuit)

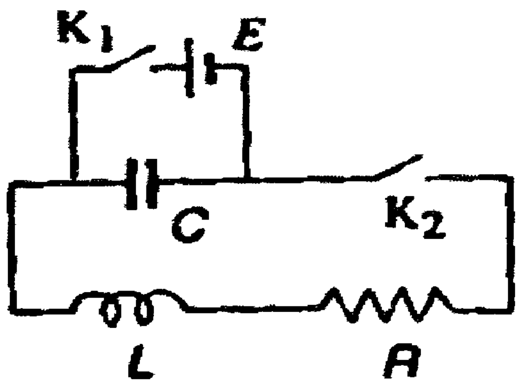


Fig.4.10

Consider a circuit containing a capacitor of capacitance C, an inductance L and resistance R joined in series (Fig. 4.10). E is a cell. K_2 is kept open. The

capacitor is charged to maximum charge Q_0 by closing the key K_1. On opening K_1 and closing key K_2, the capacitor discharges through the inductance L and resistance R. Let I be the current in the circuit and Q be the charge in the capacitor at any instant during discharge. The circuit equation then is

$$L\frac{dI}{dt} + RI + \frac{Q}{C} = 0$$

But $I = \frac{dQ}{dt}$ and $\frac{dI}{dt} = \frac{d^2Q}{dt^2}$

$$\therefore L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = 0$$

$$\frac{d^2Q}{dt^2} + \frac{R}{L}\frac{dQ}{dt} + \frac{Q}{LC} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

Let $\frac{R}{L} = 2b$ and $\frac{1}{LC} = k^2$, then

$$\frac{d^2Q}{dt^2} + 2b\frac{dQ}{dt} + k^2Q = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

The general solution of this equation is

$$Q = Ae^{\left[-b+\sqrt{(b^2-k^2)}\right]t} + Be^{\left[-b-\sqrt{(b^2-k^2)}\right]t} \quad \ldots\ldots\ldots\ldots (3)$$

Where A and B are arbitrary constants.

When t = 0, $Q = Q_0$ from Eq. (3)

$$A + B = Q_0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

$$\frac{dQ}{dt} = A\left[-b + \sqrt{(b^2-k^2)}\right]e^{\left[-b+\sqrt{(b^2-k^2)}\right]t} + B\left[-b - \sqrt{(b^2-k^2)}\right]e^{\left[-b-\sqrt{(b^2-k^2)}\right]t}$$

At t = 0, $\frac{dQ}{dt} = 0$

$$A\left[-b + \sqrt{(b^2-k^2)}\right] + B\left[-b - \sqrt{(b^2-k^2)}\right] = 0$$

$$-b(A+B) + \sqrt{(b^2-k^2)}(A-B) = 0$$

$$-bQ_0 + \sqrt{(b^2-k^2)}(A-B) = 0$$

$$\therefore A - B = -\frac{bQ_0}{\sqrt{(b^2-k^2)}} \quad \ldots\ldots\ldots (5)$$

From Eqs. (4) and (5),

$$A = \frac{1}{2}Q_0\left[1 + \frac{b}{\sqrt{(b^2-k^2)}}\right] \text{ and } B = \frac{1}{2}Q_0\left[1 - \frac{b}{\sqrt{(b^2-k^2)}}\right]$$

Putting the values of A and B in Eq. (3), we get

$$Q = \frac{1}{2}Q_0e^{-bt}\left[\left(1 + \frac{b}{\sqrt{(b^2-k^2)}}\right)e^{\sqrt{(b^2-k^2)}t} + \left(\left(1 - \frac{b}{\sqrt{(b^2-k^2)}}\right)e^{-\sqrt{(b^2-k^2)}t}\right)\right] \quad \ldots (6)$$

Case I: If $b^2 > k^2$, $\sqrt{(b^2-k^2)}$ is real and positive and the charge of the capacitor decays exponentially, becoming zero asymptotically (curve 1 of Fig. 4.11). The discharge is known as over damped, non-oscillatory or dead beat.

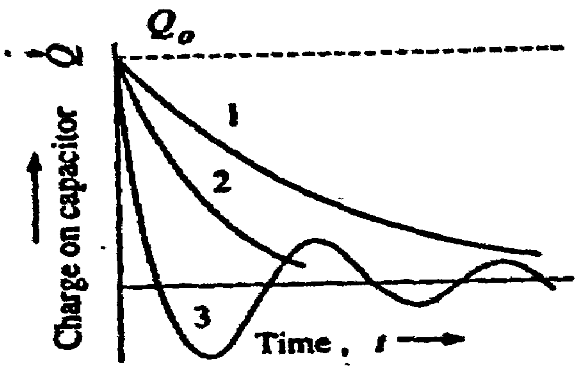


Fig. 4.11

Case II: If $b^2 = k^2$, $Q = Q_0\,(1+bt)e^{-bt}$

This represents a non – oscillatory discharge. This discharge is known as critically damped (curve 2 of Fig. 4.11). The charge decreases to zero exponentially in a short time.

Case III: If $b^2<k^2$, $\sqrt{(b^2-k^2)}$ is imaginary.

$\sqrt{(b^2-k^2)} = i\omega$ where $\omega = \sqrt{(k^2-b^2)}$

$$Q = \frac{1}{2}Q_0 e^{-bt}\left[\left(1+\frac{b}{i\omega}\right)e^{i\omega t} + \left(1-\frac{b}{i\omega}\right)e^{-i\omega t}\right]$$

$$Q = Q_0 e^{-bt}\left[\left(\frac{e^{i\omega t}+e^{-i\omega t}}{2}\right) + \frac{b}{\omega}\left(\frac{e^{i\omega t}-e^{-i\omega t}}{2i}\right)\right]$$

$$= \frac{Q_0 e^{-bt}}{\omega}(\omega cos\omega t + b sin\omega t)$$

Let $\omega = k \sin\alpha$ and $b = k\cos\alpha$ so that $\tan\alpha = \frac{\omega}{b}$.

$$Q = \frac{Q_0 e^{-bt} k}{\omega}(cos\omega t\ sin\ \alpha + cos\ \alpha\ sin\omega t)$$

Or

$$Q = \frac{Q_0 e^{-bt} k}{\omega}(\sin\,(\omega t+\alpha)$$

$$Q = \frac{Q_0 e^{-\frac{R}{2L}t}}{\sqrt{\left(\frac{1}{LC}-\frac{R^2}{4L^2}\right)}\sqrt{LC}} sin\left(\sqrt{\left(\frac{1}{LC}-\frac{R^2}{4L^2}\right)}\,t+\alpha\right) \qquad \ldots\ldots.(7)$$

This equation represents a damped oscillatory charge as shown by the curve (3). The charge oscillates above and below zero till it finally settles down to zero value.

The frequency of oscillation in the circuit is given by

$$v = \frac{\omega}{2\pi} = \frac{\sqrt{k^2-b^2}}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}}$$

When $R = 0$, $v = \frac{1}{2\pi\sqrt{LC}}$

The condition for oscillatory discharge is

$$\frac{R^2}{4L^2} < \frac{1}{LC}, \qquad or \quad R < 2\sqrt{\frac{L}{C}}$$

4.6 Determination of High Resistance by Leakage (B.G)

When a capacitor of capacitance C and initial charge Q_0 is allowed to discharge through a resistance R for a time t, the charge remaining on the capacitor is given by

$$Q = Q_0 e^{-t/CR}$$

$$\frac{Q_0}{Q} = e^{t/CR}$$

$$\log_e \frac{Q_0}{Q} = \frac{t}{CR}$$

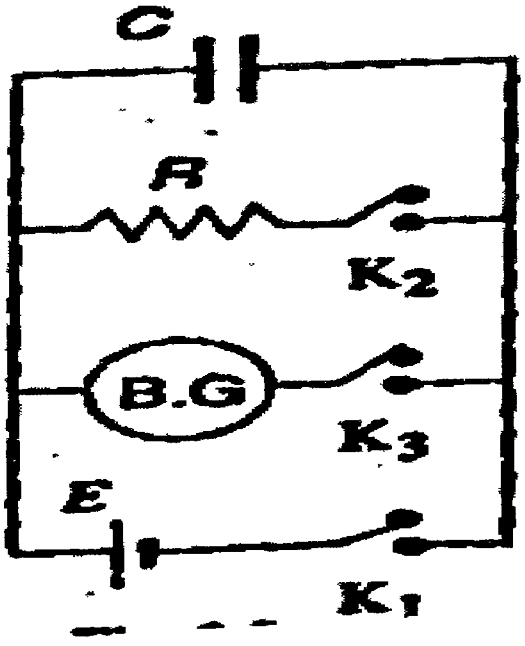


Fig.4.12

$$R = \frac{t}{C\,log_e(Q_0/Q)} = \frac{t}{2.3026C\,log_{10}(Q_0/Q)}$$

If R is high, CR will be high and the rate of discharge of capacitor will be very slow. Thus, if we determine Q_0 / Q from experiment, then R can be calculated.

Connections are made as shown in Fig. 4.12. C is a capacitor of known capacitance, R is the high resistance to be measured, B.G is a ballistic galvanometer, E is a cell, and K_1, K_2, K_3 are tap keys.

Keeping K_2 and K_3 open, the capacitor is charged by depressing the key K_1. K_1 is then opened and at once K_3 is closed. The capacitor discharges through the galvanometer which records a throw θ_0. The throw θ_0 is proportional to Q_0.

The capacitor is again charged to the maximum value keeping K_2 and K_3 open and closing K_1. K_1 is then opened and K_2 is closed for a known time, t. Some of the charge leaks through R. K_2 is opened and at once K_3 is closed. The

charge Q remaining on the capacitor then discharges through the galvanometer. The resulting throw θ is noted. Then $Q \propto \theta$

$$\text{Now, } \frac{Q_0}{Q} = \frac{\theta_0}{\theta}$$

$$\therefore R = \frac{t}{2.3026C \log_{10}(\theta_0/\theta)}$$

A series of values of t and θ are obtained. A graph is plotted between t and $\log_{10}(\theta_0 / \theta)$ which is a straight line. Its slope gives the mean value of $\frac{t}{\log_{10}(\theta_0/\theta)}$. As C is known, the value of R can be calculated.

4.7 Mean and RMS value of Alternating current

Alternating current

An alternating current is one which changes in magnitude and direction periodically.

The source of alternating emf may be a dynamo or an electronic oscillator. The alternating emf E at any instant may be expressed as

$E = E_0 \sin \omega t$ .... (1)

Here, ω is angular frequency of alternating e.m.f.

E_0 is the peak value or amplitude of alternating e.m.f.

The frequency of alternating e.m.f. $f = \frac{\omega}{2\pi}$

Time period of alternating e.m.f. $T = \frac{1}{f} = \frac{2\pi}{\omega}$

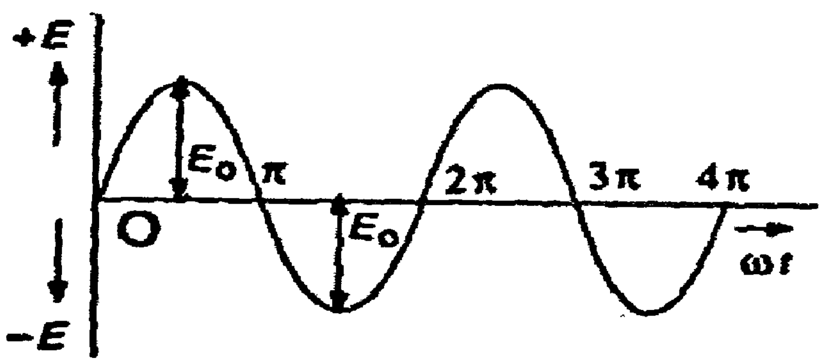


Fig.4.13

A graph of E against ωt is a sine curve (Fig. 4.13)

The corresponding current I through the circuit is given by

$I = I_0 \sin\omega t$

The alternating current in a circuit, fed by an alternating source of e.m.f. may be controlled by inductance L, resistance R and capacitance C. Due to presence of elements L and C, the current is not necessarily in phase with the applied e.m.f. Therefore alternating current is, in general, expressed as

$I = I_0 \sin (\omega t + \phi)$ .... (2)

Here φ is the phase which may be positive, zero or negative depending on the value of reactive components L and C.

Peak value of alternating current or emf: The maximum value of alternating current or emf in the positive or negative direction is called peak value of alternating current or emf. It is denoted by I_0 or E_0.

Mean value of alternating current: Mean value of alternating current is defined as its average over half a cycle.

$$I_{mean} = \frac{\int_0^{\frac{T}{2}} I dt}{\frac{T}{2}} = \frac{\int_0^{\frac{\pi}{\omega}} I_0 \sin \omega t \, dt}{\frac{\pi}{\omega}}$$

$$= \frac{I_0 \omega}{\pi} \left[\frac{-cos\omega t}{\omega}\right]_0^{\pi/\omega}$$

$$= -\frac{I_0}{\pi} [\cos \pi - \cos 0]$$

$$= \frac{2I_0}{\pi} = 0.637 \, I_0$$

Similarly, $E_{mean} = = 0.637 \, E_0$

Root mean square value of an alternating current: It is defined as the square root of the average of I^2 during a complete cycle.

$$\bar{I^2} = \frac{\int_0^{2\pi/\omega} I^2 dt}{2\pi/\omega} = \frac{\int_0^{2\pi/\omega} I_0^2 sin^2 \omega t \, dt}{2\pi/\omega}$$

$$= \frac{I_0^2 \omega}{2\pi} \int_0^{2\pi/\omega} \frac{1}{2}(1 - cos2\omega t) dt$$

$$= \frac{I_0^2 \omega}{4\pi} \left[t - \frac{\sin 2\omega t}{2\omega}\right]_0^{2\pi/\omega}$$

$$= \frac{I_0^2 \omega}{4\pi} \left[\frac{2\pi}{\omega}\right] = \frac{I_0^2}{2}$$

$$I_{rms} = \sqrt{\bar{I^2}} = \frac{I_0}{\sqrt{2}} = 0.707 \, I_0$$

Similarly, $E_{rms} = \frac{E_0}{\sqrt{2}} = 0.707 \, E_0$

The r.m.s. value of alternating current is also called as the 'effective' or the 'virtual' value of the current.

$$I_{virtual} = \frac{I_0}{\sqrt{2}} = I_{rms}$$

The r.m.s. value of alternating voltage is also called as the effective or the virtual value of the voltage.

$$E_{virtual} = \frac{E_0}{\sqrt{2}} = E_{rms}$$

Mean or average value of A.C voltage fora half cycle:

The alternating e.m.f. is represented as,

$$E_0 = E_0 \sin \omega t.$$

Mean value of alternating e.m.f. over one half cycle is,

$$E_{av} = \frac{\int_0^{T/2} E_0 \sin \omega t \, dt}{\int_0^{T/2} dt} = \frac{E_0}{T/2}\left[\frac{-\cos \omega t}{\omega}\right]_0^{T/2}$$

$$= \frac{2E_0}{T} \times \frac{T}{2\pi}\left[-\cos\left(\frac{2\pi}{T}\right)t\right]_0^{T/2}$$

$$\left(\omega = \frac{2\pi}{T}\right)$$

$$= \frac{E_0}{\pi}[-\cos \pi + \cos 0]$$

$$\therefore E_{av} = \frac{2E_0}{T}$$

R.M.S. value of alternating voltage

The R.M.S. value of an alternating e.m.f. is defined as that constant direct e.m.f. which when applied to a resistor will develop the same heat energy in it as the given alternating e.m.f. during the same interval of time.

The root mean square (r.m.s.) value of an alternating e.m.f. is the square root of the mean of the squares of the emf taken during one cycle.

Instantaneous value of the e.m.f. is

$$E = E_0 \sin \omega t.$$

Mean square value

$$= \frac{\int_0^T E_0^2 \sin^2 \omega t \, dt}{\int_0^T dt} = \frac{1}{T}\int_0^T E_0^2 sin^2 \omega t \, dt$$

$$\int_0^T E_0^2 \sin^2 \omega t \, dt = \int_0^T \frac{1}{2}(1 - \cos 2 \omega t) dt$$

$$= \int_0^T \frac{1}{2} dt - \frac{1}{2}\int_0^T \cos 2 \omega t \, dt$$

$$= \frac{1}{2}[t]_0^T - \frac{1}{2}\left[\frac{\sin 2\omega t}{2\omega}\right]_0^T$$

$$= \frac{T}{2}$$

Mean square value

$$= \frac{1}{T} \times E_0^2 \times \frac{T}{2} = \frac{E_0^2}{2}$$

$\therefore$ R.M.S. value = $E_0/\sqrt{2}$

Check your progress
5. Define peak value of an alternating current. Ans: -- --

4.8 Alternating current applied to LR and CR circuits

4.8.1 A.C. applied to LR (in series) circuits

Let an alternating emf

$$E = E_0 e^{i\omega t}$$

be applied to a circuit having an inductance L and non – inductive resistance R in series (Fig. 4.14).

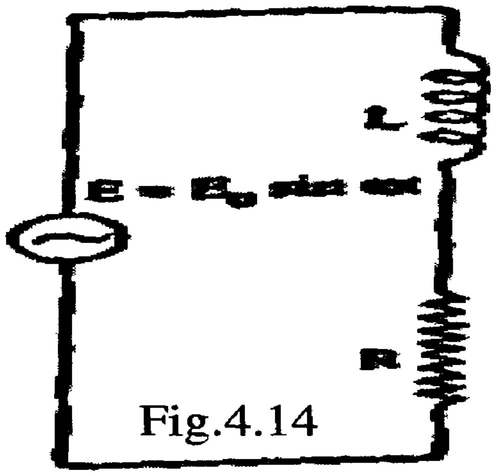


Fig.4.14

The potential drop across the inductance is

$$V_L = j\omega LI$$

The potential drop across resistance is

$$V_R = RI$$

Here, I is the current at any instant t.

$$\therefore\ E = j\omega LI + RI$$

Current in the circuit,

$$I = \frac{E}{R + j\omega L} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

$$\text{But} \quad I = \frac{E}{Z} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

Impedance of R – L circuit,

$$Z = R + j\omega L \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

$$\therefore I = \frac{E_0 e^{j\omega t}}{\sqrt{(R^2 + \omega^2 L^2)} e^{j\theta}} \left(where \tan\theta = \frac{\omega L}{R}\right)$$

$$I = \frac{E_0}{\sqrt{(R^2 + \omega^2 L^2)}} e^{j(\omega t - \theta)} \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

$$= I_0 e^{j(\omega t - \theta)} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

Here, $I_0 = \frac{E_0}{\sqrt{(R^2+\omega^2 L^2)}}$ .... (6)

It represents the peak value of the current through the circuit.

The impedance Z of the circuit is given by the term $\sqrt{(R^2 + \omega^2 L^2)}$

The current lags in phase behind the emf by an angle $\theta = \tan^{-1}\frac{\omega L}{R}$

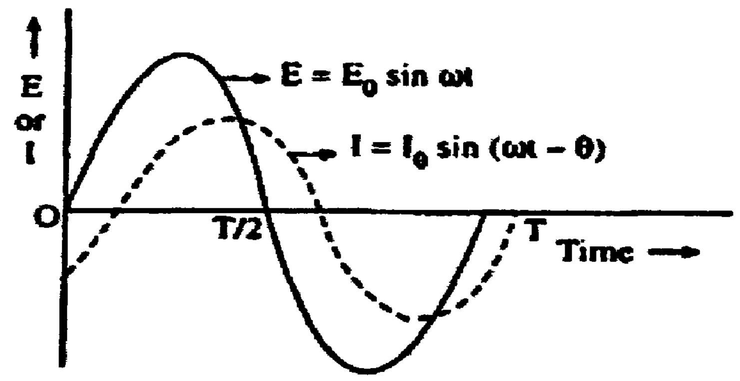


Fig. 4.15

The variation of instantaneous values of emf and current with time are represented graphically in Fig. 4.15.

4.8.2 A.C. applied to CR (in series) circuits

Let an alternating voltage $E = E_0 e^{j\omega t}$ be applied to a circuit containing a resistance R and capacitor C in series (Fig. 4.16).

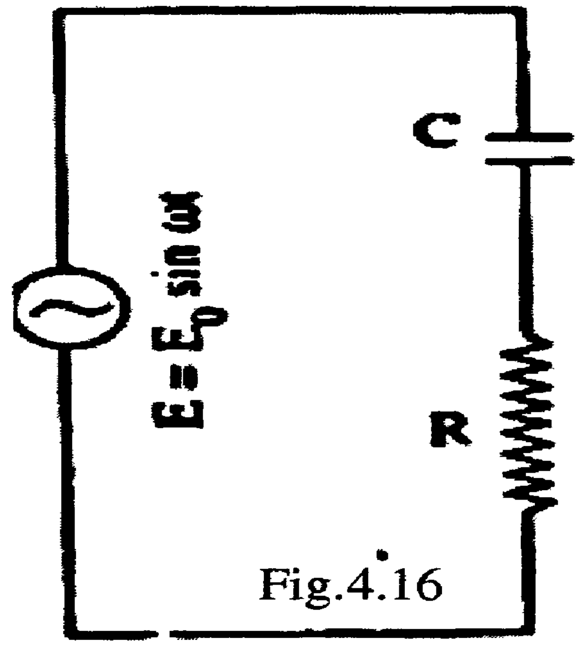


Fig.4.16

Let I be the current through the circuit and q, the charge on the plates of the capacitor at any instant. Then,

$$I\left(R + \frac{1}{j\omega C}\right) = E = E_0 e^{j\omega t} \qquad \ldots. \quad (1)$$

$$I = \frac{E}{R+\frac{1}{j\omega C}} \qquad \ldots\ldots. \quad (2)$$

$$\text{But } I = \frac{E}{Z} \qquad \ldots\ldots\ldots.. \quad (3)$$

∴ Impedance of series R – C circuit,

$$Z = R + \frac{1}{j\omega C} = R - \frac{j}{\omega C}$$

$$I = \frac{E}{R - \frac{j}{\omega C}} = \frac{E_0 e^{j\omega t}}{\sqrt{\left[\left(R^2 + \frac{1}{\omega^2 C^2}\right)\right]} e^{-j\theta}} \quad \ldots\ldots\ldots \quad (4)$$

Here, $\theta = \tan^{-1}\left(\frac{1/\omega C}{R}\right) \quad \ldots\ldots\ldots\ldots \quad (5)$

$$I = \frac{E_0}{\sqrt{\{R^2 + (1/\omega C)^2\}}} e^{j(\omega t + \theta)} \quad \ldots\ldots\ldots\ldots (6)$$

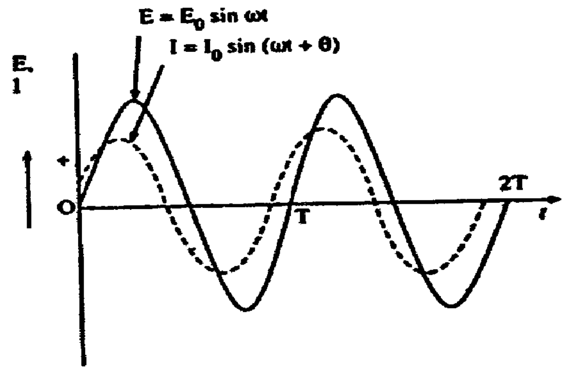


Fig.4.17

The current in this circuit thus leads the applied voltage by an angle θ (Fig. 4.17).

The impedance of R – C circuit $Z = \sqrt{\{R^2 + (1/\omega C)^2\}}$.

The actual current in the circuit is $I = I_0 \sin(\omega t + \theta)$

4.9 AC circuit containing resistance, inductance and capacitance in series (Series resonance circuit)

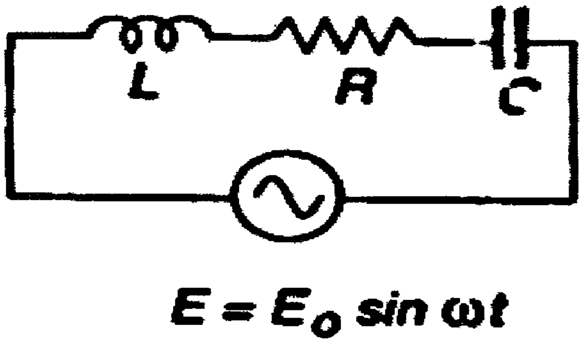


Fig.4.18

Let an alternating emf $E = E_0 \sin\omega t$ be applied to a circuit containing a resistance R, inductance L and capacitance C in series (Fig. 4.18). Let at any instant, I be the current in the circuit and Q be the charge on the capacitor.

The potential drop across the resistance = RI

The E.M.F. induced in the inductance $= L\frac{dI}{dt}$

The potential across the plates of the capacitor = Q/C

$$L\frac{dI}{dt} + RI + \frac{Q}{C} = E_0 \sin\omega t$$

Differentiating with respect to t,

$$L\frac{d^2 I}{dt^2} + R\frac{dI}{dt} + \frac{1}{C}\frac{dQ}{dt} = E_0 \omega \cos\omega t \quad \ldots\ldots\ldots \quad (1)$$

Let the trial solution be of the form

$I = I_0 \sin(\omega t - \phi)$ .... (2)

Here, I_0 and ϕ are constants to be determined.

$$\frac{dI}{dt} = I_0 \omega \cos(\omega t - \theta)$$

And

$$\frac{d^2 I}{dt^2} = -I_0 \omega^2 \sin(\omega t - \theta)$$

Substituting these values of I, $\frac{dI}{dt}$ and $\frac{d^2 I}{dt^2}$, in Eq. (1) we get

$$-LI_0 \sin(\omega t - \phi) + RI_0\, \omega \cos(\omega t - \phi) + \frac{I_0}{C} \sin(\omega t - \phi) = E_0 \omega \cos \omega t$$

Or

$$\left(-L\omega^2 + \frac{1}{C}\right) I_0 \sin(\omega t - \phi) + R\omega I_0 \cos(\omega t - \phi)$$
$$= E_0 \omega[\cos(\omega t - \phi) + \phi]$$
$$= E_0 \omega[\cos(\omega t - \phi) \cos \phi - \sin(\omega t - \phi) \sin \phi]$$

Equating the coefficients of sin (ωt -ϕ) and cos (ωt -ϕ) on either side,

$\left(-L\omega^2 + \frac{1}{C}\right) I_0 = -E_0 \omega \sin \phi$ .... (3)

And $R\omega I_o = E_0 \omega \cos\phi$ .... (4)

$$\tan \phi = \frac{\left(-L\omega^2 + \frac{1}{C}\right)}{R\omega} = \frac{\omega L - \frac{1}{C\omega}}{R} ....(5)$$

Squaring and adding Eqs. (3) and (4), we get

$$I^2{}_0 \left[\left(-L\omega^2 + \frac{1}{C}\right)^2 + R^2 \omega^2\right] = E_0^2 \omega^2$$

Or $$I_o^2 \left(R^2 + L\omega - \frac{1}{C\omega}\right)^2 = Eo^2$$

$$I_o = \frac{E_o}{\sqrt{\left[\left(L\omega - \frac{1}{C\omega}\right)^2 + R^2\right]}}$$ .... (6)

Substituting the value of I_o in Eq. (2), we get

$$I = \frac{E_o}{\sqrt{\left[\left(L\omega - \frac{1}{C\omega}\right)^2 + R^2\right]}} \sin(\omega t - \phi)$$ ...... (7)

Here $\phi = \tan^{-1}\left(\frac{L\omega - \frac{1}{C\omega}}{R}\right)$

Eq. (7) represents the current at any instant.

The quantity $\sqrt{\left[\left(L\omega-\frac{1}{C\omega}\right)^2+R^2\right]}$ is the impedance Z of the circuit. Lω and 1/ωC respectively represent inductive reactance X_L and capacitive reactance Xc.

$$Z=\sqrt{[R^2+(X_L-X_C)^2]}$$

The current lags in phase behind emf by an angle

$$\phi=\tan^{-1}\left(\frac{L\omega-\frac{1}{C\omega}}{R}\right)=\tan^{-1}\frac{X_L-X_C}{R}$$

The following three cases arise:

1. When X_L>X_C, Φ is positive, so that the current lags behind the applied emf.

2. When X_L> X_C, Φ is negative, so that the current leads the applied emf.

3. When X_L> X_C, Φ = 0, and the current is in phase with the emf.

Series Resonance Circuit

The value of current at any instant in a series LCR circuit is given by

$$I=\frac{E_o}{\sqrt{\left[\left(L\omega-\frac{1}{C\omega}\right)^2+R^2\right]}}\sin(\omega t-\phi) \quad ..... \quad (1)$$

Here, $\sqrt{\left[\left(L\omega-\frac{1}{C\omega}\right)^2+R^2\right]}=Z$

is called the impedance of the circuit.

At a particular frequency, $\omega L=\frac{1}{\omega C}$, the impedance becomes minimum. It is given by Z = R. This particular frequency ν_o at which the impedance of the circuit becomes minimum and, therefore the current becomes maximum, is called the resonant frequency of the circuit.Such a circuit which admits maximum current is called series resonance circuit.

Thus at ν_o, we have

$\omega L=\frac{1}{\omega C}$, Or $2\pi\nu_0 L=\frac{1}{2\pi\nu_0 C}$ or $\nu_0=\frac{1}{2\pi\sqrt{LC}}$

The maximum current in the circuit = $I_o = E_o/R$

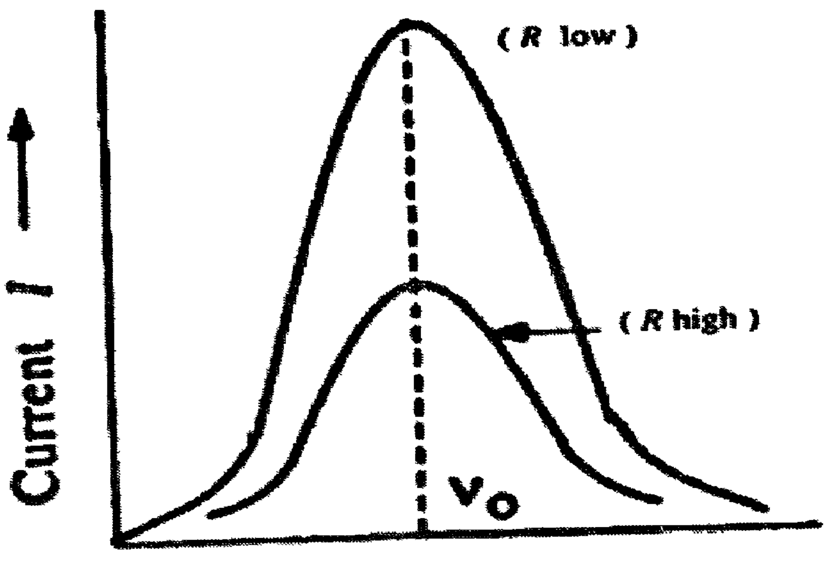


Fig.4.19

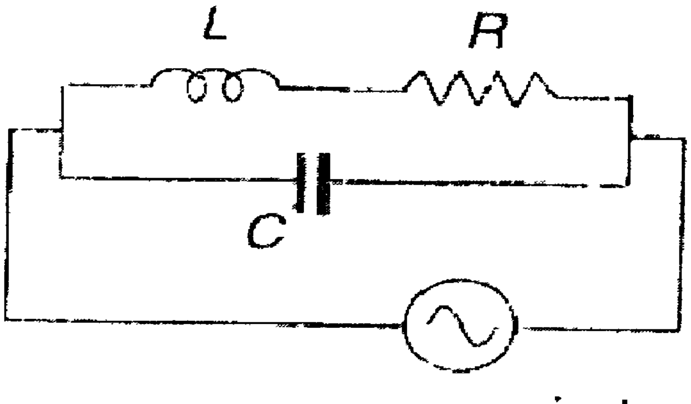
The variation of current with frequency of applied voltage is shown in Fig.4.19. The sharpness of peak depends upon the resistance R of the circuit. For low resistance, the peak is sharp.

Acceptor Circuit: The series resonance circuit is often called an 'acceptor' circuit. By offering minimum impedance to currents at the resonant frequency, it is able to select or accept most readily the current of this one frequency from among those of many frequencies.

In radio receivers, the resonant frequency of the circuit is tuned (by changing C) to the frequency of the signal desired to be detected.

10 AC circuit containing resistance, inductance and capacitance in Parallel (Parallel resonance circuit)

Parallel Resonance Circuit



$\underline{E} = E_0\, e^{j\omega t}$

Fig. 4.20

Here, capacitor C is connected in parallel to the series combination of resistance R and inductance L. The combination is connected across the AC source (Fig.4.20) .The applied voltage is sinusoidal, represented by

$$E = E_0\, e^{j\omega t}$$

Complex impedance of L – branch

$Z_1 = R + jL\omega$

Complex impedance of C – branch

$Z_2 = \frac{1}{j\omega C}$

Z_1 and Z_2 are in parallel

$$\frac{1}{Z} = \frac{1}{R + j\omega L} + \frac{1}{1/j\omega C} = \frac{1}{R + j\omega L} + j\omega C$$

$$= \frac{R - j\omega L}{(R + j\omega L)\times(R - j\omega L)} + j\omega C$$

$$= \frac{R}{R^2 + (L\omega)^2} + j\left[\omega C - \frac{L\omega}{R^2 + (L\omega)^2}\right]$$

The current

$$I = \frac{E}{Z} = E \times \frac{1}{Z}$$

$$I = E\left(\frac{R}{R^2 + (L\omega)^2} + j\left[\omega C - \frac{L\omega}{R^2 + (L\omega)^2}\right]\right)$$

Let

$$A\cos\phi = \frac{R}{R^2 + (L\omega)^2};\ A\sin\phi = \omega C - \frac{L\omega}{R^2 + (L\omega)^2}$$

$$I = E(A\cos\phi + jA\sin\phi) = EAe^{i\phi} = EoAe^{j(\omega t + \phi)}$$

Here

$$\phi = \tan^{-1}\frac{\left(C\omega - \frac{L\omega}{R^2 + (L\omega)^2}\right)}{\left(\frac{R}{R^2 + (L\omega)^2}\right)}$$

$$A^2 = \frac{R^2}{R^2 + (L^2\omega^2)^2} + \left(C\omega - \frac{L\omega}{R^2 + (L^2\omega^2)^2}\right)^2$$

The magnitude of the admittance

$$Y = \frac{1}{Z} = \frac{\sqrt{R^2 + \left(\omega CR^2 + \omega^3 L^2 C - \omega L\right)^2}}{R^2 + L^2\omega^2}$$

The admittance will be minimum, when

$\omega CR^2 + \omega^3 L^2 C - \omega L = 0$

or

$$\omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$\nu_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

This is the resonant frequency of the circuit.

If R is very small so that $\frac{R^2}{L^2}$ is negligible compared to $\frac{1}{LC}$

$$\nu_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

At a minimum admittance, i.e., maximum impedance, the circuit current is minimum.

The graph between current and frequency is shown in Fig.4.21.

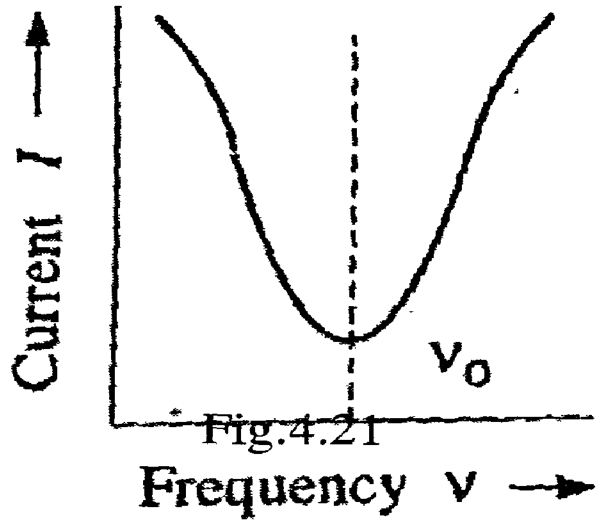


Fig.4.21

Impedance at resonance

$$Z = \frac{R^2 + (L\omega)^2}{R}$$

But $R^2 + (L\omega)^2 = \frac{L}{C}$ at resonance.

$$Z = \frac{L}{C}$$

Thus smaller the resistance R, larger is the impedance.

If R is negligible, the impedance is infinite at resonance.

Rejector Circuit

The parallel resonant circuit does not allow the current of the same frequency as the natural frequency of the circuit. Thus it can be used to suppress the current of this particular frequency out of currents of many other frequencies. Hence this circuit is known as a 'rejector' or 'filter' circuit.

Comparison between Series and Parallel Resonance Circuit

The behavior of a parallel resonant circuit is strikingly different from that of a series resonant circuit. In both cases, the impedance is resistive but whereas parallel resonance implies maximum impedance, series resonance implies minimum impedance.

	Series Resonant Circuit		Parallel Resonant Circuit
1.	An acceptor circuit	1.	A rejector circuit
2.	Resonant frequency $\nu_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$	2.	Resonant frequency $\nu_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$
3.	At resonance the impedance is a minimum equal to the resistance in the circuit.	3.	At resonance the impedance is a maximum nearly equal to infinity.
4.	Selective	4.	Selective
5.	Used in the tuning circuit to separate the unwanted frequency from the wanted frequency from the incoming frequencies by offering low impedance at that frequency.	5.	Used to present a maximum impedance to the wanted frequency, usually in the plate circuit of valves.

Check your progress

6. What is called impedance?

Ans: --

4.11 Power in an AC circuit

Consider an ac circuit containing resistance, inductance and capacitance. E and I vary continuously with time. Therefore power is calculated at any instant and then its mean is calculated over a complete cycle.

The instantaneous values of the voltage and current are given by

$$E = E_o \sin \omega t$$

$$I = I_o \sin(\omega t - \phi)$$

Here Φ is the phase difference between current and voltage.

Hence power at any instant is

$$E \times I = E_o I_o \sin \omega t \sin(\omega t - \phi)$$

$$= \frac{1}{2} E_o I_o [\cos \phi - \cos(2\omega t - \phi)] \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

Average power consumed over one complete cycle is

$$P = \frac{\int_0^T EI dt}{\int_0^T dt}$$

$$P = \frac{\int_0^T \frac{1}{2} E_o I_o [\cos \phi - \cos(2\omega t - \phi)] dt}{T}$$

$$= \frac{E_o I_o}{2T} \left| (\cos \phi) t - \frac{\sin(2\omega t - \phi)}{2\omega} \right|_0^T$$

$$= \frac{E_o I_o}{2T} \left[(\cos \phi) T - 0 - \frac{\sin(2\omega T - \phi)}{2\omega} + \frac{\sin(-\phi)}{2\omega} \right]$$

Now $T = \frac{2\pi}{\omega}$ and Sin (4π-φ) = Sin (-φ)

$$P = \frac{1}{2} \frac{E_o I_o \omega}{2\pi} \left| (\cos \phi) \frac{2\pi}{\omega} - \frac{\sin(-\phi)}{2\omega} + \frac{\sin(-\phi)}{2\omega} \right|$$

$$= \frac{1}{2} E_o I_o \cos \phi$$

$$= \frac{E_o}{\sqrt{2}} \frac{I_o}{\sqrt{2}} \cos \phi$$

$$= E_{r.m.s} I_{r.m.s} \cos \phi \qquad (2)$$

Average power = (Virtual volts) x (Virtual amperes x cos φ

The term (Virtual volts) x (Virtual amperes) is called apparent power and cos ϕ is called the power factor. Thus

True power = apparent power / power factor.

Evidently, the power factor is the ratio of the true power to the apparent power.

As cos φ is the factor by which the product of the rms values of the voltage and current must be multiplied to give the power dissipated, it is known as the 'power factor' of the circuit. For a circuit containing resistance, capacitance and inductance in series,

$$\tan\varphi = \frac{\omega L - \frac{1}{\omega C}}{R}$$

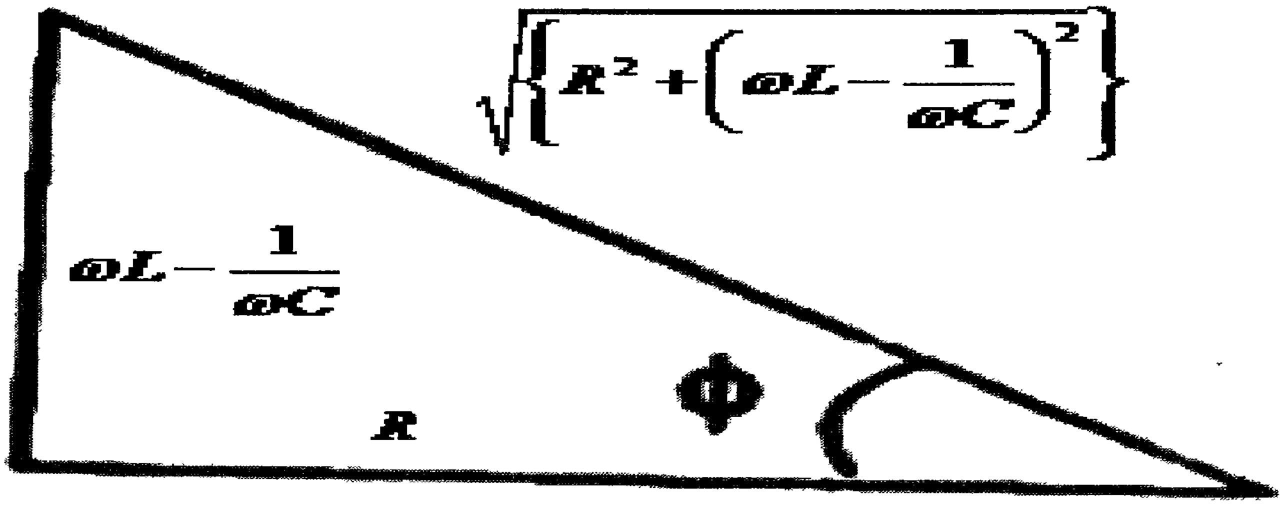


Fig.4.22

From figure 4.22. the expression for the power factor is

$$\cos\phi = \frac{R}{\sqrt{\left\{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2\right\}}}$$

Special cases:

(1) In a purely resistive circuit, φ=0 or cos φ= 1.

$\therefore truepower = E_v \times I_v$.

(2) In a purely inductive circuit, current lags behind the applied emf by 90^0 so that

Φ=90° or cosΦ=0.

(3)In a purely capacitive circuit, current leads the applied emf by 90^0 so that

Φ= -90° or cos(-90°)=cos 90°=0.

$\therefore truepower = 0$

(4)In an ac circuit containing a resistance and inductance in series, power factor

$$\cos\phi = \frac{R}{\sqrt{R^2+(L\omega)^2}}.$$

(5)In an ac circuit containing a capacitance C and a resistance R in series,

$$\cos\phi = \frac{R}{\sqrt{R^2+\frac{1}{C^2\omega^2}}}.$$

4.12 Wattless current

The average power dissipated during a complete cycle is $E_v\ I_v \cdot \cos\Phi$

The current in A.C. circuit is said to be wattles when the average power consumed in the circuit is zero. If an ac circuit is purely inductive or purely capacitive with no ohmic resistance, phase angle $\Phi = \pi/2$ so that $\cos\Phi = 0$ or the power consumed is zero. The current in such a circuit does not perform any useful work and is rightly called the wattless or idle current. In this situation, the circuit does not consume any power, through it offers a resistance to the flow of alternating current in it. It is the principle of choke coil.

4.13 Q – factor

Check your progress
7. What is called power factor?
Ans: --

$$\text{Q-Factor} = \frac{\text{Reactance of the coil at resonance}}{\text{Resistance of the circuit}} = \frac{L\omega_o}{R}$$

Q- Factor determines the degree of selectivity of the circuit while tuning. This is because, for larger values of Q- factor the frequency response curve of the circuit is a steep narrow peak. For smaller values of Q- factor, the frequency response curve is quite flat (Fig.4.23)

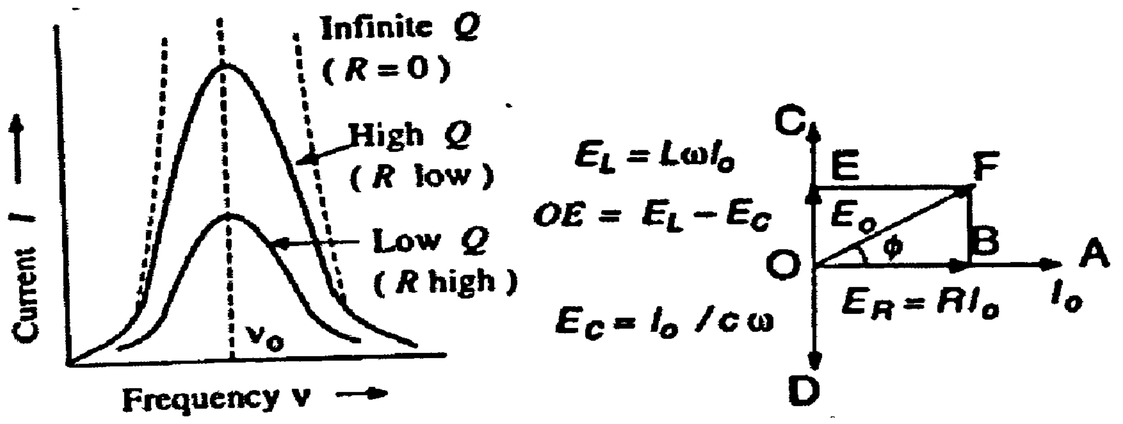


Fig. 4. 23

Vector Diagram. A vector diagram of a series LCR circuits is shown in fig4.23 .Since L-C-R is connected in series, the current through each is same. Let E_R, E_L and E_C be the potential drops across resistance, inductance and capacitance.

The vector $E_R = I_O R$ is in phase with the current.

The vector $E_L = \omega\ L\ I_O$ is 90^0 advance of the current.

The vector $E_c = I_O / \omega\ C$ is 90^0 behind the current

Let the vectors OB, OC and OD represent E_R, E_L, and E_C. If $E_L > E_C$, the resultant of these two is $(E_L - E_C)$. This is represented by OE.

$$OE = OC - OD$$

$$= I_o\left(L\omega - \frac{1}{C\omega}\right);\ ; where L\omega > \frac{1}{C\omega}$$

$$= [OB^2+BF^2]^{1/2} = [OB^2+OE^2]^{1/2}$$

$$E_o = \left[Io^2R^2 + I_o^2\left(L\omega - \frac{1}{C\omega}\right)^2\right]^{1/2}$$

$$= Io\left[\left(L\omega - \frac{1}{C\omega}\right)^2 + R^2\right]^{1/2}$$

$$Io = \frac{E_o}{\sqrt{\left[\left(L\omega - \frac{1}{C\omega}\right)^2 + R^2\right]}}$$

The current lags behind the applied voltage by Φ given as

$$\phi = \tan^{-1}\left(\frac{L\omega - \frac{1}{C\omega}}{R}\right)$$

4.14 Choke

A choke coil is an inductance coil which is used to control the current in an ac circuit.

Construction:

A choke consists of a coil of several turns of insulated thick copper wire of low resistance but large inductance, wound over a laminated core (Fig.4.24).

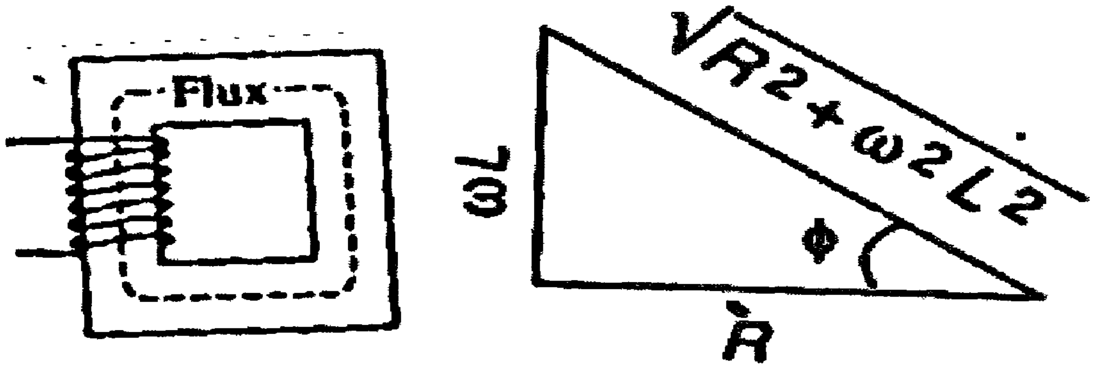


Fig. 4.24

The core is layered and is made up of thin sheets of stalloy to reduce hysteresis losses. The laminations are coated with shellac to insulate and bound together firmly so as to minimize loss of energy due to eddy currents.

The average power dissipated in the choke coil is given by

$$P = \frac{1}{2}E_oI_o\cos\phi$$

If the resistance of the choke coil is R and the inductance of the choke coil is L, then the power factor cos Φ is given by

$$\cos\phi = \frac{R}{\sqrt{(R^2 + \omega^2 L^2)}}$$

The inductance L of the choke coil is quite large on account of its large number of turns and the high permeability of iron core, while its resistance R is very small. Hence cos Φ is nearly zero. Therefore, the power absorbed by the coil is extremely small. Thus the choke coil reduces the strength of the current without appreciable wastage of energy. The only waste of energy is due to the hysteresis loss in the iron core. The loss due to eddy currents is minimized by making the core laminated.

Preference of choke coil over an ohmic resistance for diminishing the current.

The current in an AC circuit can also be diminished by using an ordinary ohmic resistance (rheostat) in the circuit. But such a method of controlling AC is not economical as much of the electrical energy (I^2Rt) supplied by the source is wasted as heat. Hence the choke coil is to be preferred over the ohmic resistance.

The energy used in establishing the magnetic fields in the choke coil is restored when the magnetic field collapses. Hence to regulate ac, it is more economical to use a choke than a resistance.

Choking coils are very much used in electronic circuits, mercury lamps and sodium vapour lamps.

A.F. Choke

Chokes used in low frequency a.c. circuit have an iron core so that the inductance may be high. These chokes are known as audio-frequency (A.F) chokes (Fig.4.25a).

Fig. 4.25 (a)

Fig. 4.25 (b)

R.F. Choke

For radio frequencies, air chokes are used since a low inductance is sufficient. These are called radio frequency (R.F) or high frequency (H.F) chokes and are used in wireless receiver circuits (Fig.4.25 b).

Check your progress
8. Define choke coil. Ans: -- --

4.15 Skin effect

The current density J remains constant over any given section when a steady current passes through a uniform wire. But, when, an alternating current of high frequency flows through a wire, the current density is not uniform at all points across a section. There is a greater current density at the surface layers than at the interior layers of the conductor. When the frequency is very large, the current is almost entirely confined to the surface layer. This phenomenon is called Skin effect. Since alternating currents of high frequency do not pass through the entire cross-section of the wire, the effective resistance of a wire for A.C. is much greater than that for D.C. Hence the conductor required to carry high frequency alternating currents consists of a number of strands of fine wire connected in parallel at their ends, and insulated throughout their length from each other. This increases the surface area and thus decreases the resistance. Such a conductor has negligible Skin effect and it's A.C. resistance is very nearly equal to its D.C. resistance.

Explanation:

When the current passes along the axis of a cylindrical wire, the magnetic flux is finite. When the same current passes through the surface of the wire, the magnetic flux within the wire is zero. The current is changing with time with a fixed frequency over its entire cross-section. Hence the rate of change of flux in the wire (and the emf induced) is greater due to a current near the axis than that for a current near the periphery. As the induced e.m.f opposes the applied e.m.f, the effective resistance or impedance is higher at the centre than in the outer layers. Hence less current passes through the inner layers than through the outer ones. Since the reactance is dependent on frequency ($X_L = \omega L = 2\pi\upsilon L$) the magnitude of the effect is greater at higher frequencies.

4.16 AC bridges

Introduction

The Wheatstone bridge principle is also applicable to AC networks. The only modification is that here complex impedances and currents are used instead of resistances. The null point is determined with the help of a pair of head phones.

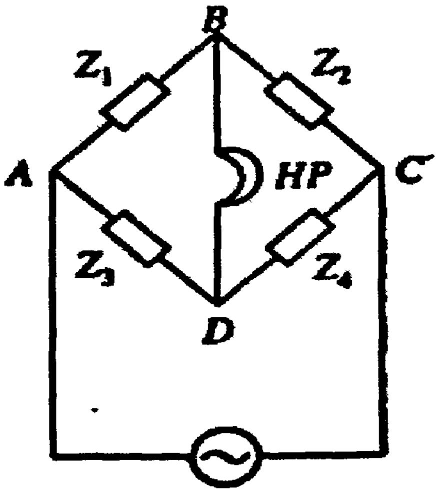


Fig. 4.26

At balance, the points B and D are at the same potential and the head phone HP gives a minimum sound. The condition of balance for the bridge shown in Fig.4.26 is given by

$$\frac{Z_1}{Z_2} = \frac{Z_3}{Z_4}$$

Here Z_1, Z_2, Z_3 and Z_4 are the vector impedances of the branches. The phase balance condition is adjusted by changing reactances (L and C) in the arm.

4.17 Owen's Bridge

This bridge is used to measure the inductance of a coil in terms of resistance and capacitance. The bridge circuit is shown in Fig. 4.27. C1 and C2 are standard capacitors. R_1 and R_4 are variable resistances. R_4 is a fixed resistance. The coil whose self-inductance L is to be determined is connected in the arm AD in series with R_3.

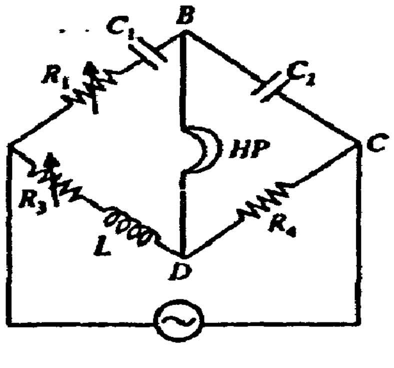


Fig. 4.27

Let the impedances in the four arms of the bridge be Z_1, Z_2, Z_3 and Z_4 at the balance point.

$$Z_1 = R_1 + \frac{1}{jC_1\omega};\ Z_2 = \frac{1}{jC_2\omega};\ Z_3 = R_3 + jL\omega;\ Z_4 = R_4$$

When the bridge is balanced, we have

$$\frac{Z_1}{Z_2} = \frac{Z_3}{Z_4}$$

Or $\quad Z_1Z_4 = Z_2Z_3$

$$\left(R_1 + \frac{1}{jC_1\omega}\right)R_4 = \frac{1}{jC_2\omega}(R_3 + jL\omega)$$

$$R_1R_4 + \frac{R_4}{jC_1\omega} = \frac{R_3}{jC_2\omega} + \frac{L}{C_2}$$

Equating the imaginary parts we have

$$\frac{R_4}{C_1} = \frac{R_3}{C_2} \quad \text{or} \quad \frac{R_3}{R_4} = \frac{C_2}{C_1} \qquad (1)$$

Equating the real parts,

$$R_1R_4 = \frac{L}{C_2}$$

$L = R_1R_4C_2$ (2)

Thus we have two balances conditions which are independent of one another. The condition (1) is satisfied by varying R_3 and (2) by varying R_1 till the sound is minimum in both the cases.

4.18 Determination of self-inductance by Anderson's method

The experiment is performed in two stages.

(a) D.C. Balance

The circuit connections are shown in Fig.4.28 (a). The given coil of self-inductance L and resistance S is connected in arm DC. The ratio arms P and Q are fixed to ratio 1:1. The resistance R is adjusted for balance. This gives the approximate value of resistance S of the coil. The experiment is repeated by making the P: Q ratio to be 10:1 and 100:1. The accurate value of D.C. resistance of the coil is found by the Wheat stone's bridge relation

$$\frac{P}{Q} = \frac{R}{S} \quad or \quad S = R\frac{Q}{P}$$

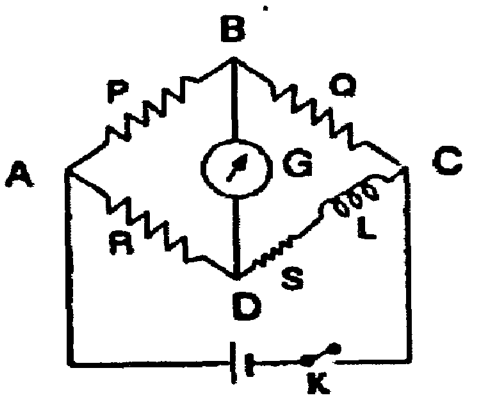


Fig. 4.28 (a)

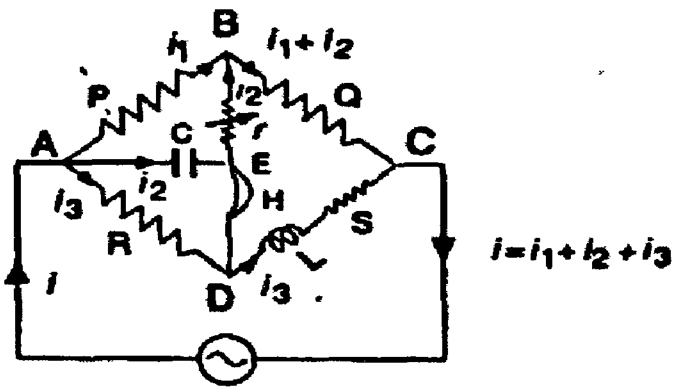


Fig. 4.28 (a)

(b) A.C. Balance

An ac source (oscillator) is connected between A and C (Fig.4.28 b). A variable non-inductive resistance r is connected in series with a capacitance C and this combination is connected in parallel with the arm AB. A headphone H is

connected between E and D. The resistance r is adjusted until minimum sound is heard in the headphone. The value of L is calculated using the formula,

$$L = C[RQ + r(R+S)]$$

Theory

Let the instantaneous currents in the different arm be as shown in Fig. 4. 28 (b)

At the time of balance, (i.e. no current through headphone)

Potential at E = Potential at D

Applying Kirchhoff's II law, we have

(i) For mesh ABEA,

$$i_1 P - i_2 (r + \frac{1}{j\omega C}) = 0$$

Or $$i_1 = \frac{1}{P}\left(r + \frac{1}{j\omega C}\right) i_2 \qquad (1)$$

(ii) For mesh AEDA,

$$\frac{i_2}{j\omega C} - i_3 R = 0$$

Or $$i_3 = \frac{1}{j\omega CR} i_2 \qquad (2)$$

(iii) For mesh BCDB,

$$(i_1 + i_2)Q - i_3(S + j\omega L) + i_2 r = 0$$

Or $$i_1 Q + i_2(Q + r) - i_3(S + j\omega L) = 0 \qquad (3)$$

Here $\frac{1}{j\omega C}$ and jωL are the impedances offered by capacitor C and inductance L respectively. ω is the angular frequency of applied a.c.

Substituting Eqns. (1) and (2) in Eq. (3), we get

$$\frac{Q}{P}\left(r + \frac{1}{j\omega C}\right) i_2 + i_2(Q + r) - \frac{S + j\omega L}{j\omega CR} i_2 = 0$$

$$\frac{Q}{P}\left(r + \frac{1}{j\omega C}\right) + (Q + r) - \frac{S + j\omega L}{j\omega CR} = 0 \qquad (4)$$

Equating the real and imaginary parts separately to zero, we get

$$\frac{Q}{P} r + Q + r - \frac{L}{CR} = 0 \qquad (5)$$

And $$\frac{Q}{P\omega C} - \frac{S}{\omega CR} = 0 \qquad (6)$$

Eq. (6) gives $\frac{P}{Q}=\frac{R}{S}$ $or\ S=R\frac{Q}{P}$ (7)

This is condition for D.C balance

From Eq. (5), we get

$$\frac{L}{CR}=\frac{Q}{P}r+Q+r$$

Or $L=CR\left(\frac{Q}{P}r+Q+r\right)$

Or $L=C\left(R\frac{Q}{P}r+RQ+Rr\right)$

Or $L=C\left(Sr+RQ+Rr\right)$ from Eq. (7)

Or $L=C[RQ+r(S+R)]$

4.19 Maxwell's Bridge

The circuit of Maxwell's L-C bridge for the determination of self-inductance is shown in Fig. 4.29. The coil whose inductance L is to be determined is placed in the arm CD, in series with a variable non-inductive resistance R_4. Arms AB, BC and AD contain non-inductive resistances R_1, R_2 and R_3 respectively. A standard variable capacitor C_1 is connected in parallel with resistance R_1 in the arm AB.

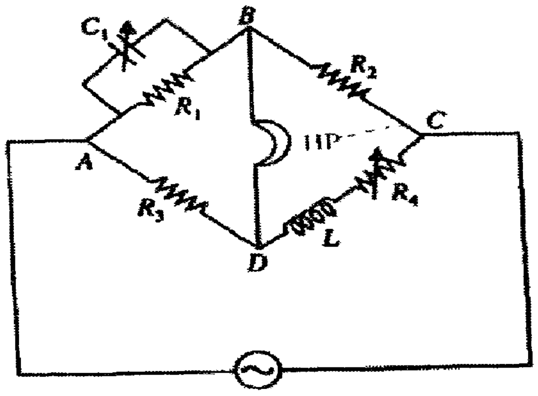


Fig.4.29

Let Z_1, Z_2, Z_3 and Z_4 be the impedances of branches AB, BC, AD and DC respectively.

Here $\frac{1}{Z_1}=\frac{1}{R_1}+\frac{1}{1/jC_1\omega}=\frac{1}{R_1}+jC_1\omega$; Z1= $\frac{R_1}{\left(1+j\omega C_1R_1\right)}$

$Z_2=R_2$; $Z_3=R_3$; $Z_4=R_4+jL\omega$

According to the condition of balance,

$$\frac{Z_1}{Z_2}=\frac{Z_3}{Z_4}$$

$$\frac{R_1}{(1+j\omega C_1R_1)R_2}=\frac{R_3}{(R_4+jL\omega)}$$

$$R_1R_4+jL\omega R_1=R_2R_3+R_2R_3R_1j\omega C_1$$

Equating real and imaginary parts, we get

$$R_4R_1=R_2R_3$$

$$\frac{R_1}{R_2}=\frac{R_3}{R_4}$$

$$L=R_2R_3C_1$$

Eqs. (1) and (2) are the two balance conditions which are independent of one another. When using the bridge, R_1/R_2 is kept a simple ratio. R_4 is varied until the head phone gives minimum sound. Now the potentials at B and D are equal in magnitude. Then the capacitance C_1 is varied until the sound in the headphone reduces to a further minimum. The experiment may be repeated varying the values of the resistances.

4.20 Let us sum up

- Growth of current in a circuit containing a resistance and inductance

  $$I=I_o\left(1-e^{-\left(\frac{R}{L}\right)t}\right)$$

- Decay of current in a circuit containing a resistance and inductance

  $$I=I_oe^{-\left(\frac{R}{L}\right)t}$$

- Growth of charge in CR circuit

  $$Q = Q_0\ (1 - e^{-\frac{t}{CR}})$$

- Decay of charge in CR circuit

  $$Q = Q_0e^{-t/CR}$$

- Growth of charge in a circuit with inductance, capacitance and resistance

  $$Q = Q_0 - \frac{1}{2}Q_0e^{-bt}\left[\left(1+\frac{b}{\sqrt{(b^2-k^2)}}\right)e^{\sqrt{(b^2-k^2)}t} + \left(\left(1-\frac{b}{\sqrt{(b^2-k^2)}}\right)e^{\sqrt{(b^2-k^2)}t}\right)\right]$$

- Determination of High Resistance by Leakage (B.G)

  $$R=\frac{t}{2.3026C\ log_{10}(\theta_{10}/\theta)}$$

- The maximum value of alternating current or emf in the positive or negative direction is called peak value of alternating current or emf. It is denoted by I_0 or E_0.
- The root mean square (r.m.s.) value of an alternating e.m.f. is the square root of the mean of the squares of the emf taken during one cycle.

- The current in A.C. circuit is said to be wattles when the average power consumed in the circuit is zero.
- Q-Factor $= \frac{Reactance\ of\ the\ coil\ at\ resonance}{Resistance\ of\ the\ circuit} = \frac{L\omega_o}{R}$

  Q- Factor determines the degree of selectivity of the circuit while tuning.
- A choke coil is an inductance coil which is used to control the current in an ac circuit.

4.21 Unit – end exercises

1. Derive the Helmholtz equations for the growth and decay of current in a circuit having L and R.
2. Obtain an expression for the growth and decay of charge in a capacitor through a resistance.
3. Describe the method of measuring high resistance by the leakage method.
4. Distinguish between the mean value and RMS value of an alternating current.
5. Discuss the LCR series resonance circuit with necessary theory.
6. Investigate the behavior of a resonance circuit consisting of C in parallel with L and R.
7. Find an expression for the power in an AC circuit. Determine the condition that the current in the circuit may be wattless.
8. Give the theory of choke coil.
9. What is meant by skin effect?
10. What is (i) power factor and (ii) Q – factor.
11. Describe the method of determining the self-inductance of a coil using Owen's bridge with theorem.
12. Explain Maxwell's AC bridge method for finding self-inductance.
13. Describe Anderson's bridge method for the evaluation of coefficient self-induction of a coil.

4.22 Problems for discussion

1. An e.m.f of 10 volt is applied to a circuit having a resistance of 10 Ω and an inductance of 0.5 henry. Find the time required by the current to obtain 63.2% of its final value. What is the time constant of the circuit?
2. Show that capacitive time constant τ_c has the dimensions of time.

3. A charged capacitor of capacitance 0.01 μF is made to discharge through a circuit consisting of a coil of inductance 0.1 henry and an unknown resistance. What should be the maximum value of the unknown resistance, if the discharge of the capacitor is to be oscillatory?
4. If the charge of a capacitor of capacitance 2 μF in leaking through a high resistance of 100 mΩ is reduced to half its maximum value, calculate the time of leakage.
5. Why is shock from ac more severe than that from dc?
6. An alternating potential of 100 volt and 50 hertz is applied across a series circuit having an inductance of 5 henry, a resistance of 100 Ω and a variable capacitance. At what value of capacitance will the current in the circuit be in phase with the applied voltage? Calculate the current in this condition. What will be the potential differences across the resistance, inductance and capacitance?
7. What is the power expended in a series LCR circuit at resonance?
8. An electric lamp which runs at 100 volt D.C. and 10 ampere current are connected to 220 volt 50 Hz A.C. mains. Calculate the inductance of the choke?

4.23 Answers to check your progress and problems for discussion

To check your progress:

1. The time constant L/R of an L-R circuit is the time taken by the current to grow from zero to 0.632 times the steady maximum value of current in the circuit.
2. $I = I_0 e^{-(R/L)\,t}$
3. I set: L = 1 mH, R = 1000 Ω; II set : L = 10 mH, R = 10 Ω. The first set is suitable because for it, the time constant L/R is smaller.
4. The time constant of a CR circuit is defined as the time taken by the current to fall from maximum to 0.368 of its maximum value.
5. The maximum value of alternating current or emf in the positive or negative direction is called peak value of alternating current or emf. It is denoted by I_0 or E_0.
6. In any circuit the ratio of the effective voltage to the effective current is defined as the impedance Z of the circuit.
7. The term (Virtual Volts X Virtual amperes) is called apparent power and cos Φ is called the power factor. Thus

   True power = apparent power X power factor.

   Evidently, the power factor is the ratio of the true power to the apparent power.

8. A choke coil is an inductance coil which is used to control the current in an ac circuit.

To Problems:

1. $I = I_0\left(1 - e^{-Rt/L}\right)$

Given

$$\frac{I}{I_0} = \frac{63.2}{100}; \frac{R}{L} = \frac{10}{0.5} = 20$$

$$\frac{63.2}{100} = 1 - e^{-20t}$$

$$e^{-20t} = 1 - 0.632 = 0.368$$

$$e^{20t} = \frac{1}{0.368} = 2.717$$

$$20t = \log_e 2.717$$

$$t = \frac{1}{20} X\, 2.3026 X \log_{10} 2.717$$

$$= \frac{2.3026 X\, 0.4341}{20} = 0.05 \text{ Sec}$$

The time constant if the circuit is

$$\frac{L}{R} = \frac{0.5}{10} = \frac{1}{20} \text{ Sec}$$

2. $\tau_c = CR$. Now Capacitance $C = \frac{Q}{V} = \frac{It}{V} = \frac{t}{(V/I)} = \frac{t}{R}$

$$\tau_c = \frac{t}{R} \times R = t$$

Therefore τ_c has the dimensions of time.

3.If R is the maximum value of the resistance for the discharge to be oscillatory, then

$$\frac{R^2}{4L^2} = \frac{1}{LC} \text{ or } R = \sqrt{\frac{4L}{C}} = \sqrt{\frac{4 \times 0.1}{0.01 \times 10^{-6}}} = 6324\ \Omega$$

$$R = \frac{1}{2.3026\ C \log_{10}(Q_0/Q)}$$

4.Here, $C = 2 X 10^{-6}$ F, $R = 10^8 \Omega$, $Q_0/Q = 2$

$t = 2.3026\ CR \log_{10}(Q_0/Q) = 2.3026\ (2 \times 10^{-6})\ 10^8 \log_{10}2$

$= 138.7$ s

5.The peak voltage in an ac circuit is given by $E_o = \sqrt{2}E_v$. With an average voltage supply of 220 volts, the peak voltage is $E_o = \sqrt{2} \times 220 \cong 331 volts$.

Thus, an ac supply of 220 volts varies between values equal to $\pm 311 volts$. Hence; the shock from an ac is more severe.

6.For resonance, $\omega L = \frac{1}{\omega C}$ or $c = \frac{1}{\omega^2 L}$

Here, $\omega = 2\pi\nu = 100\pi, L = 5H$

$$c = \frac{1}{(100\pi)^2 \times 5} = 2\times10^{-6} farad = 2\mu F$$

The current at resonance = $I_{rms} = \frac{E_{rms}}{R} = \frac{100}{100} = 1.0A$

$$P.D. across R = I_{rms} \times R = 1.0\times100 = 100 Volt(rms)$$

$$P.D. across L = I_{rms} \times \omega L = 1.0\times(100\pi)\times5 = 1570 Volt(rms)$$

$$P.D. across C = I_{rms} \times \frac{1}{C\omega} = 1.0\times\frac{1}{(2\times10^{-6})\times100\pi}$$

=1570 Volt (rms)

7. At resonance, we have $\phi = 0$ because $L\omega = \frac{1}{C\omega}$

The current is then in phase with the emf. The mean power I given by

$$\overline{P} = E_{r.m.s.} I_{r.m.s} \cos\phi = E_{r.m.s.} I_{r.m.s}$$

8.Resistance of the lamp $R = \frac{V}{I} = \frac{100}{10} = 10\Omega$.

If the lamp is to be run from 220 volts, 50 Hz A.C. mains a choke (inductance) should be placed in series with the lamp in order to increase its effective resistance.

Let L be the inductance of the required choke. Then

$$\text{Impedance} = \sqrt{(R^2 + \omega^2 L^2)} = \sqrt{[(10)^2 + (2\pi\times50)^2 L^2]}$$

$$= \sqrt{(100 + 10^4\pi^2 L^2)}.$$

$$current = \frac{Voltage}{impedance}$$

$$10 = \frac{220}{\sqrt{(100 + 10^4\pi^2 L^2)}}$$

L = 0.062 H

4.24 Suggested readings

1. Electricity and Magnetism – Prantosh Chakraborty, I Edition, Reprint 2008, New Age International Private Limited Publisher.
2. Electricity and Magnetism – P.L.Sehgal, K.L.Chopra, N.K.Sehgal, VI Edition, Reprint 2009, S.Chand& Sons, New Delhi.

Unit V: Magnetism

Structure

5.1 Introduction
5.2 Objectives
5.3. Magnetic Induction (B):
5.4. Magnetic Materials
5.5 Magnetisation
5.6. Hysteresis
5.7. Work done in taking unit volume of a magnetic material through a complete cycle of magnetisation
5.8 Area of the hysteresis loop
5.9. Experiment to draw B-H curve (Ballistic method):Circuit Description:
5.10. Gouy's method for measurement of Susceptibility
5.11. Maxwell's Equations In Material Media
5.12. Types of Currents
5.13. Displacement current
5.14. Magnitude of Displacement current
5.15. Significance of Displacement current
5.16. Maxwell's equations in material media
5.17. Plane Electromagnetic Waves in free Space – Velocity of Light
5.18. Propagation of Electromagnetic wave through a homogeneous, isotropic dielectric medium (non-conducting isotropic medium):
5.19. Let us sum-up
5.20. Unit –end exercises
5.21. Problems for discussion
5.22. Answers for Check your progress and problems for discussion
5.23. Answers for problem for discussion
5.24. Answers for problem for discussion

5.1. Introduction:

The magnetic properties of a substance are explained in terms of tiny current loops within the substance. These current loops arise due to motion of electrons within atom.

We know that an atom consists of positively charged nucleus, surrounded by a cloud of electrons. These electrons circulate about the nucleus in definite orbits and also spin about their own axes. These moving electrons are equivalent to tiny current loops and produce magnetic fields.

5.2 Objectives

After going through this unit, you will be able to:

- define Magnetic induction, Magnetising field, Magnetisation and susceptibility
- distinguish the properties dia, para and ferromagnetic materials.
- describe Ballistic method experiment to draw B-H curve
- explain Hysteresis curve and its importance
- discuss the properties of ferrites
- describe Guoy's method for the determination of susceptibility
- derive Maxwell's equations and its significance.

5.3. (i) Magnetic Induction (B)

If a positive test charge q moving with velocity V through a point in a magnetic field experience a force F, then the magnetic induction B at that point is defined by

$$\mathbf{F} = q\,(\mathbf{v} \times \mathbf{B})$$

The magnitude of the magnetic induction is thus defined by

$$B = \frac{F}{q(v \sin\theta)}.$$

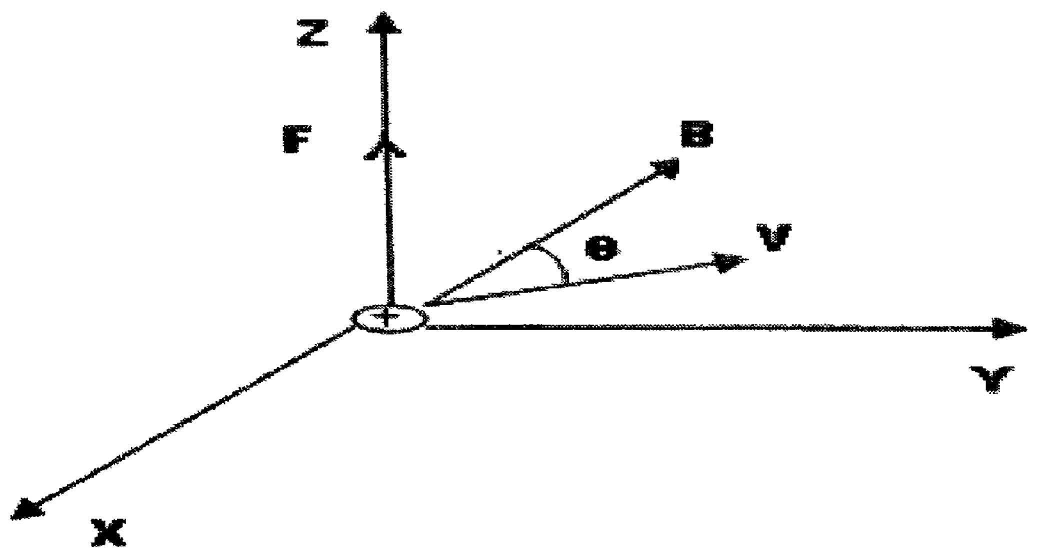


Fig 5.1

Here, θ is the angle between **V** and **B** (Fig 5.1) **SI** unit of **B** is Weber / metre^2

The magnetic field can be represented by 'lines of induction'. The tangent to the line of induction at any point gives the direction of **B**. The number of lines of induction per unit, area normal to their direction is equal to the magnitude of **B.**

Fig.5.2 represents a bar of soft iron placed in vacuum with its length parallel to the lines of magnetising field. The bar is magnetised and the number of lines within the specimen (**B**) is greater than the number of lines in free (**H**) i.e. **B>H** in iron.

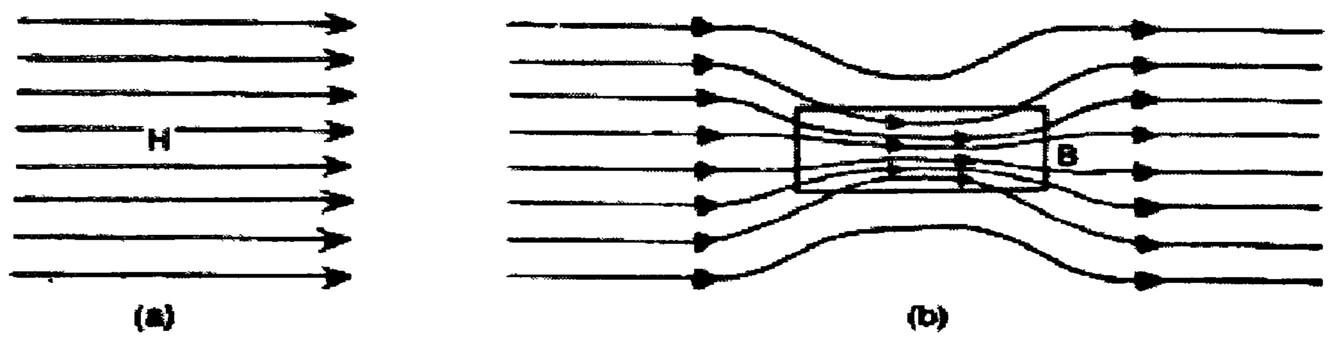


Fig 5.2

Definition of Magnetic Induction (B)

The net number of magnetic lines of force passing per unit area normally within the material medium when it is placed in an external magnetic (or magnetising) field is called the magnetic induction (**B**) within the sample.

(ii)Magnetising field (H):

When a magnetic material is placed in a magnetic field, it becomes magnetised. The capability of the magnetic field to magnetise a materials is expressed by means of magnetic vector **H**, called the magnetising field or magnetic field intensity.

Definition: Magnetising field is defined as the force experienced by a unit north pole placed at a point in the magnetic field.

It direction is the same as that of **B.**

It unit is Am^{-1}

(iii) Magnetisation (M):

When a magnetic material is placed in a magnetic field, the elementary current – loops in the material become aligned parallel to the field. The material is then magnetised, and acquires a magnetic dipole moment.

Definition: Magnetisation M of the material is defined as the magnetic dipole moment induced per unit volume of the material. Unit of **M** is Am^{-1}.

Let **m** be the magnetic dipole moment of a specimen of volume V.

Then $$M = \frac{m}{V}$$

In an unmagnified matter **M** will be zero. In a uniformly magnetized matter, each atomic magnetic dipole will point in the same direction and magnetization **M** will be constant throughout.

Relation between the three magnetic vectors, B, H and M:

Consider a Rowland ring having a toroidal winding of N turns around it. When a current i_0 is sent through the winding, the ring is magnetised along its circumferential length. The current i_0 is the real current which magnetises the ring. This magnetisation arises due to the alignment of the elementary current-loops (magnetic dipoles) resulting from electronic motions in the material. Fig **5.3** shows a section of the magnetised ring. The small circles

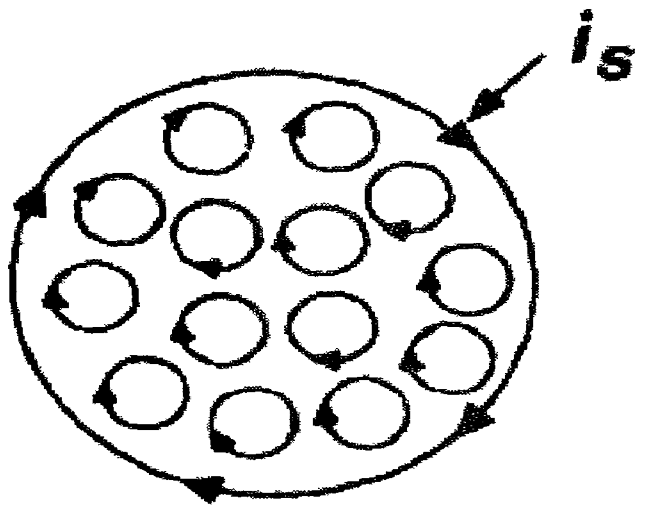


Fig 5.3

represent the current-loops. These internal tiny circular electron currents tend to cancel each other due to the fact that adjacent currents are in opposite directions. As such, there is no net current inside the core. The current in the outer portions of the outer-most loops remain uncancelled. The numerous tiny localized surface currents can be replaced by a single closed current i_s along the surface. Such a current is called **Amperian current.**

Let A= area of cross section and

l=circumferential length of the ring. Then, volume of the ring = lA

The ring behaves like a large dipole of magnetic moment i_s A.

Magnetisation = M=magnetic moment per unit volume.

$$= \frac{i_s A}{lA} = \frac{i_s}{l}$$

The magnetisation M, therefore, is the surface current per unit length of the ring. This is commonly called **magnetisation current**.

Now, the magnetic induction B within the material of the ring arises due to the free current i_0 in the winding, as well as due to the magnetisation of the ring itself which can be described in terms of Amperian surface current.

$$B = \mu_0\left(\frac{Ni_0}{l} + \frac{i_s}{l}\right) = \mu_0\left(\frac{Ni_0}{l} + M\right) \qquad \left(\frac{i_s}{l} = M\right)$$

Here $\frac{Ni_0}{l}$ is the free current per unit length and $\frac{i_s}{l}$ is the Amperian surface current per unit length.

$$\frac{B}{\mu_0} - M = \frac{Ni_0}{l}$$

The quantity $\frac{B}{\mu_0} - M$ is called magnetising field or magnetic field intensity H. i.e

$$H = \frac{B}{\mu_0} - M$$

or

$$B = \mu_0(H + M)$$

In vector form $\boldsymbol{B} = \mu_0(\boldsymbol{H} + \boldsymbol{M})$

This is the relation between the three vectors **B, H and M.**

Magnetic susceptibility:

For isotropic linear Para - and diamagnetic materials it is found experimentally that the magnetisation M is proportional to the magnetic field intensity H. That is,

$$M \propto M \text{ or } M = x_m H$$

The constant x_m is called the magnetic susceptibility of the materia . I may be defined as the ratio of the magnetisation M to the magnetic field intensity H.

$$x_m = \frac{M}{H}$$

Definition: The magnetic susceptibility of a material is defined as the intensity of magnetisation acquired by the material for unit field strength.

We can classify magnetic materials in terms of susceptibilityx_m. If x_m is positive, the material is called paramagnetic. If x_m is negative, the material is diamagnetic. The characteristic of ferromagnetic materials is that x_m is positive and very large. However, in ferromagnetic materials, M is not exactly proportional to H, and so x_m is not a constant. M may even be finite when H=0.

Magnetic Permeability:

Consider the relation

$$B = \mu_0(H + M)$$

$$=\mu_0(H + x_m M)$$

$$= \mu_0(1 + x_m)H$$

$$= \mu H$$

Here, $\mu = \mu_0$ $(1+x_m)$, is called the magnetic permeability of the material

Magnetic permeability (μ) of a medium is defined as the ratio of magnetic induction to the intensity of the magnetising field. i.e., $\mu = \frac{B}{H}$; (For vacuum $x_m = 0$ *and* $\mu = \mu_0$)

Hence magnetic induction in vacuum is $B_0 = \mu_0 H$.

The ratio $\frac{B}{B_0} = \frac{\mu}{\mu_0}$ is called the relative permeabilityμ_r. Obviously $\mu_r = 1+x_m$

We may also classify magnetic materials in terms of the relative permeability μ_r.

Diamagnetism: $\mu_r < 1$ $(For\ Bi, \quad \mu_r = 1 - 0.00017)$

Para magnetism: $\mu_r > 1$ $(For\ Al, \quad \mu_r = 1 + 0.00002)$

Ferromagnetism: $\mu_r \gg 1 (For\ pure\ Fe, \mu_r = 200{,}000)$

Check your progress
1. Define Magnetic permeability 2. The magnetic susceptibility of the medium is 2.14 x 10^{-11} H/m. Calculate its relative permeability Ans: -- --

5.4. Magnetic Materials

a) Properties of Diamagnetic materials

The substances which get weakly magnetised by a strong magnetic field in a direction opposite to that of the applied magnetic field are called **diamagnetic substances**.

The examples of diamagnetic substances are **copper, silver, gold, bismuth, antimony, quartz, diamond, sodium, water, alcohol.**

(i) For diamagnetic substances, relative permeability μ_r is slightly less than 1.

(ii) When a bar of diamagnetic substance is placed in strong magnetic field, the magnetism is induced in a direction opposite to the external field (Fig 5.4). So the total numbers of magnetic lines of induction are less in the material than that in free space. This shows that the magnetic dipole moment, intensity of magnetisation (M) and magnetic susceptibility are negative.

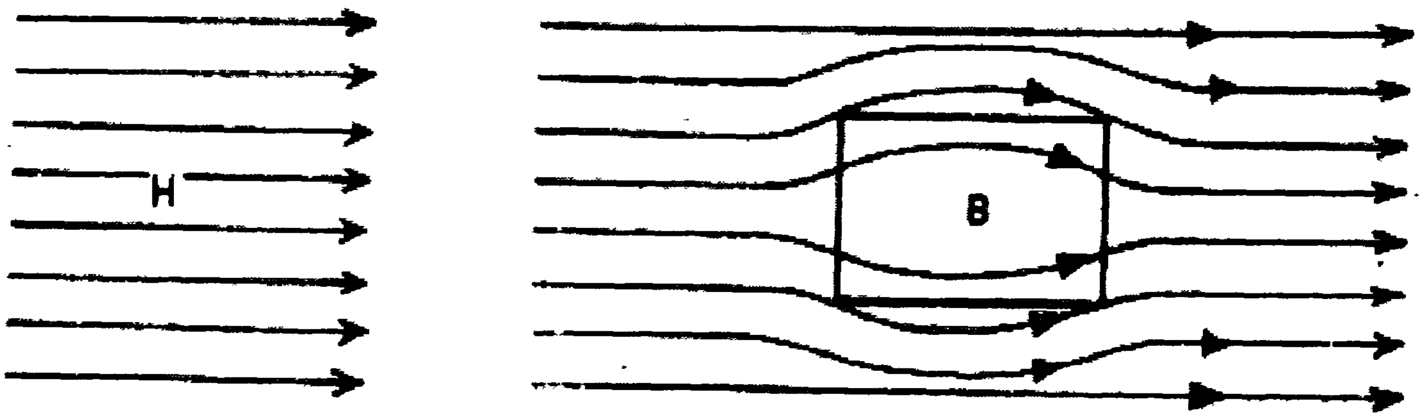


Fig 5.4

(iii) The susceptibility (χ_m) of a diamagnetic material is independent of temperature.

(iv) If a bar of a diamagnetic substance is suspended between the poles of a magnet, it stays at right angles to field direction (Fig 5.5).

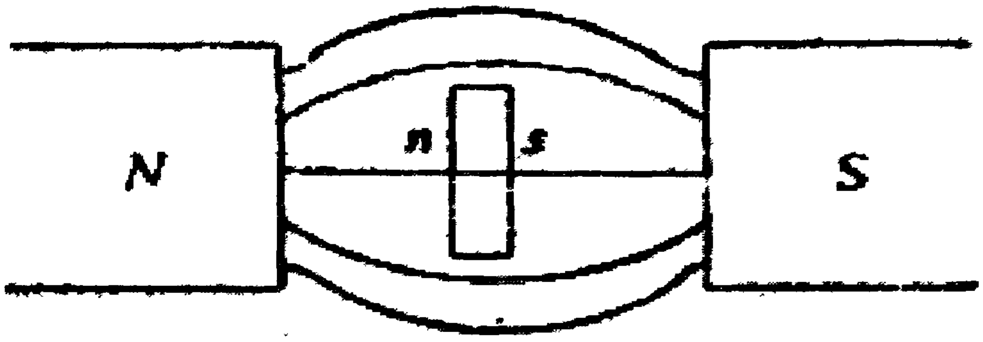


Fig 5.5

(v) In a non-uniform magnetic field, a diamagnetic substance tends to move from the stronger to weaker parts of the field.

A diamagnetic liquid is placed in a watch glass resting on two pole pieces very near to each other (Fig 5.6)

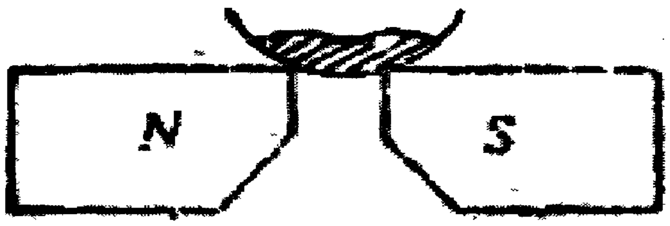


Fig 5.6

The liquid accumulates on the sides where the field is weaker, producing a depression in the middle.

Fig 5.7

The reverse effect is observed when the poles are far apart (Fig 5.7)

(vi) A diamagnetic liquid shows a depression in the limb of U- tube placed between pole pieces of a strong magnet.

(vii) A diamagnetic gas when allowed to ascend in between the poles of a magnet spreads across the field.

b) Properties of paramagnetic materials:

The substances which get weakly magnetised by a strong magnetic field of the same direction as the applied field, are called **paramagnetic substances.**

The examples of paramagnetic substances are aluminium, magnesium, titanium, tungsten, chromium, manganese, liquid oxygen, air, copper sulphate and solutions of salts of iron and nickel.

(i) For paramagnetic substances, the relative permeability μ_r is slightly greater than 1.

(ii) When a bar of paramagnetic substance is place in a strong magnetic field, the magnetism is induced parallel to the direction of external field (Fig 5.8). So the total numbers of magnetic lines of induction within the material are slightly greater than that in free space. This shows that magnetic dipole moment, intensity of magnetisation (M) and magnetic susceptibility are positive but small.

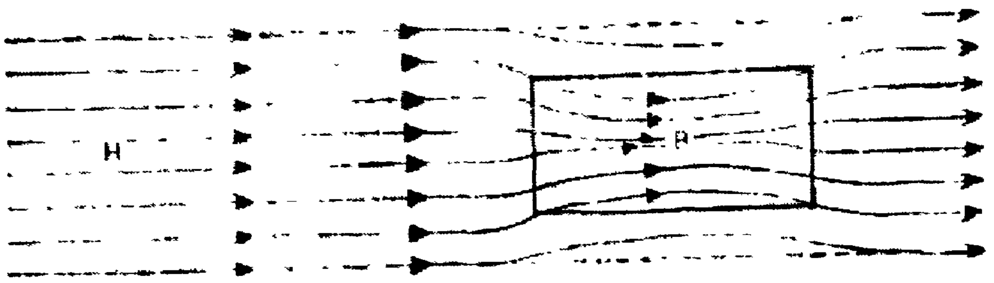


Fig 5.8

(iii) The susceptibility (χ_m) of a paramagnetic material decreases with rise in temperature.

(iv) If a bar of paramagnetic substance is suspended between poles of a magnet, it stays parallel to the lines of force (Fig 5.9)

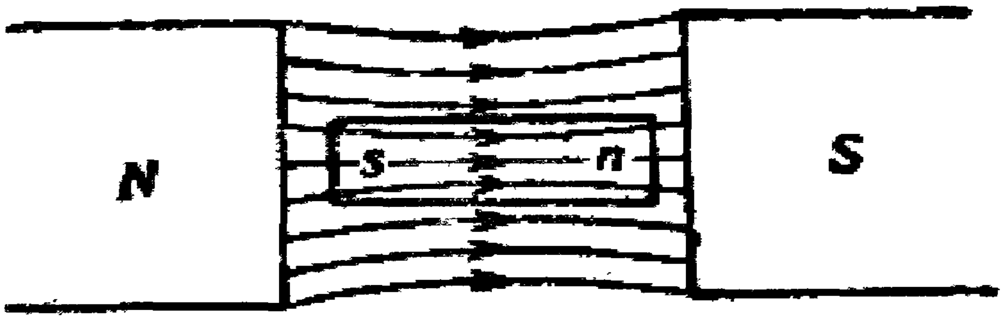


Fig 5.9

(v) In a non-uniform field, paramagnetic substances tend to move from weaker to stronger parts f the magnetic field.

A paramagnetic liquid is placed in a watch glass resting on two pole pieces very near to each other (Fig 5.10)

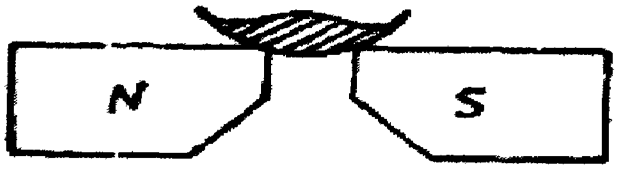


Fig 5.10

The liquid accumulates in the middle where the field is strongest.

If the pole-pieces are far apart, the field is strongest near the poles.

The liquid moves away from the centre producing a depression in the middle (Fig 5.11)

Fig 5.11

(vi) A paramagnetic liquid shows a rise in the limb of U-tube placed between pole pieces of a strong magnet (fig 5.12).

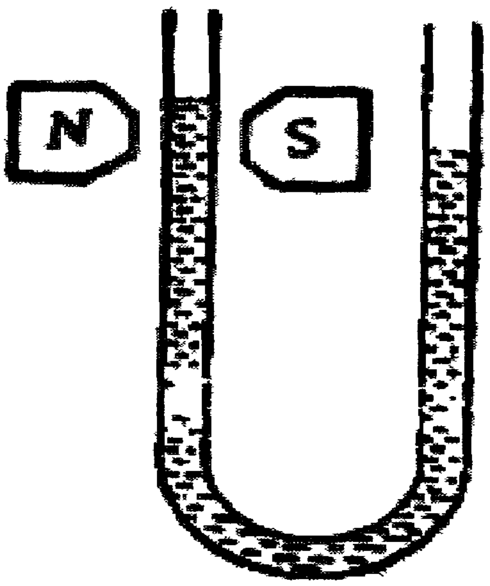


Fig 5.12

(vii) A paramagnetic gas when allowed to ascend between the pole pieces of a magnet spreads along the field.

c) Properties of ferromagnetic materials:

Ferromagnetic substances are those which are strongly magnetised by relatively weak magnetic field and in the same sense as the applied magnetic field.

The examples are: Iron (Fe), Nickel (Ni), Cobalt (Co), steel, gadolinium and their alloys, Magnetite (Fe_3O_4). Etc.

(i) Ferromagnetic show all properties of paramagnetic to a much high degree. For example, if a bar of ferromagnetic material is placed in an external magnetic field, strong magnetism is induced parallel to the direction of field. So the total numbers of magnetic lines of force within the material are much greater than that in free space. This shows that magnetic dipole moment, intensity of magnetisation (M) and magnetic susceptibility (x_m) are positive and quite large. The relative permeability μ_r of these materials is very large.

(ii) They get strongly magnetised in the direction of the external field and so they are strongly attracted by magnets.

(iii) They set themselves parallel to the external field if suspended freely.

(iv) This materials exhibit the phenomenon of hysteresis.

(v) As temperature increases, the value of magnetic susceptibility (x_m) decreases. Above a certain temperature, known as Curie temperature, ferromagnetic become paramagnetic

Check your progress
3. Does every magnet necessarily have a north and south pole 4. What are ferromagnetic substances? Ans: -- -- -- ------------------------------------

5.5 Magnetisation

The paramagnetic, diamagnetic and ferromagnetic behaviour of substances can be explained in an elementary way in terms of the electron theory of matter.

Each electron is supposed to be revolving in an orbit around the nucleus. Each moving electron behaves like a tiny current loop and therefore possesses orbital magnetic dipole moment. Furthermore, each electron is spinning about an axis through itself. This spin also gives rise to a magnetic dipole moment. In general, the resultant magnetic dipole moment of an atom is the vector sum of the orbital and spin magnetic dipole moments of its electrons.

Explanation of Diamagnetism.

Diamagnetism occurs in those substances whose atom consists of an even number of electrons. The electrons of such atoms are paired. The electrons in each pair have orbital motions as well as spin motions in opposite sense. The resultant magnetic dipole moment of the atom is thus zero. Hence when such a substance is placed in magnetic field, the field does not ten do to align the atoms (dipoles) of the substance. The field, however, modifies the motion of the electrons in orbits which are equivalent to tiny current loops. The electron moving in a direction fields are slowed down, while the other is accelerated (Lenz's law). The electron pair, and hence the atom, thus acquire an effective magnetic dipole moment which is opposite to the applied field. Hence for diamagnetic materials M is opposite to h. So the susceptibility (x_m) of a diamagnetic substance is negative and is very small.

Explanation of Paramagnetism:

In paramagnetic materials, the magnetic fields associated with the orbiting and spinning electrons do not cancel out. There is a net intrinsic moment in it. The molecules in it behave like little magnets. When such a substance is placed in an external field it will turn and line up with its axis parallel to the external field. Thus it tends to move further into the field. i.e., there is force of attraction. The diamagnetic force of repulsion is also present, but it is not so strong as the attracting force arising from the magnetic properties of the material. Since M and B (and hence H) are in the same direction in paramagnetic, the susceptibility (x_m) is positive. When a paramagnetic substance is heated, the thermal agitation of its atoms increase. So the alignment of the dipoles becomes more difficult. This is why the magnetisation of paramagnetic substances decreases as the temperature of the substance increases. (x_m) $\propto V_1$

Explanation of Ferromagnetism:

Ferromagnetic substances are very strongly magnetic. The best-known examples of ferromagnets are the transition metals Fe, Co, and Ni. A ferromagnetic has a spontaneous magnetic moment – a magnetic moment even in zero applied fields. The atoms (or molecules) of ferromagnetic materials have a net intrinsic magnetic dipole moment which is primarily due to the spin of the electrons. The interaction between the neighbouring atomic magnetic dipoles is very strong. It is called **spin exchange interaction** and is present even in the absence of an external magnetic field. It turns out that the energy of two neighbouring atomic magnets due to this interaction is the least when their magnetic moments are parallel. The neighbouring magnetic moments are,

therefore, strongly constrained to take parallel orientation (Fig 5.13). This effect of the exchange interaction to align the neighbouring magnetic dipole moments parallel to one another spreads over a small finite volume of the bulk. This small (1-0.1 mm across) volume of the bulk is called a **domain** (Fig.5.14). All magnetic moments within a domain will point in the same direction, resulting in a large magnetic moment. Thus the bulk material consists of many domains. The domains are oriented in different directions. The total magnetic moments of a sample of the substance is the vector sum of the magnetic moments of the component domains.

Fig 5.13

Fig 5.14

In an unmagnified piece of ferromagnetic material, the magnetic moments of the domains themselves are not aligned. When an external field is applied, those domains that are aligned with the field increase in size at the expense of the others. In a very strong field, all the domains, are lined up in the direction of the field and provide the high observed magnetisation.

If a ferromagnetic material is heated to a very high temperature, the thermal vibrations may become strong enough to offset the alignment within a domain. At such temperature, the material loses its ferromagnetic property and behaves like a paramagnetic material. The critical temperature above which a ferromagnetic material becomes a paramagnetic is called the **Curie temperature**.

3.0 Check your progress

Q: How the pigeon be able to sense the direction?

A: The pigeon may well be able to sense the direction because of a built-in magnetic "compass" with in their skull that are connected with a large number of nerves to the pigeon brain.

5.6. Hysteresis

Consider an unmagnified ferromagnetic substance (say iron bar) in a magnetising field. The intensity of magnetisation (M) induced is measured for different values of magnetising field H. The variation f M with variation in H is shown in Fig.5.15.

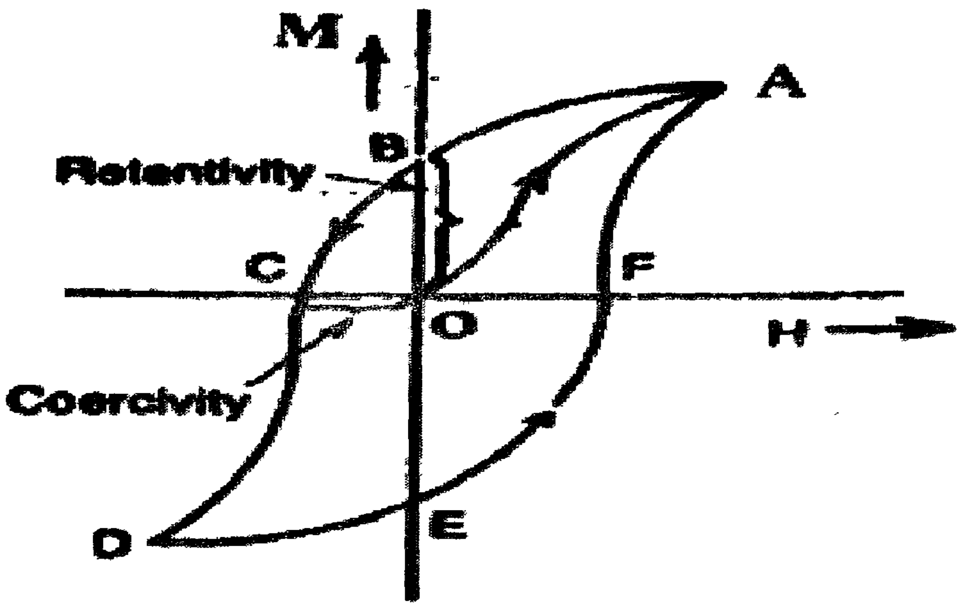


Fig 5.15

The point O represents an initially unmagnified specimen and a zero magnetic intensity.

As H is increased from zero, the value of M also increases as in OA. At point A, the intensity of magnetisation M becomes constant i.e., it has acquired its saturation value.

Now consider that the magnetising field H is decreased. It is obvious from the figure that intensity of magnetisation also decreases without following path AO. It follows the path AB. At B, the intensity of magnetisation has some value while magnetising field H is zero. The value of intensity of magnetisation for which H=0 is called **retentivity or residual magnetism.**

Definition of Retentivity. The retentivity or remnance is defined as the intensity of magnetisation remaining in the substance when the magnetic field has been reduced to zero.

Further consider that direction of H is reversed. In this case, the curve BCD is obtained. For the part BC, the value of M decreases as the value of H is increased in reverse direction up to a point C where M is zero. This shows that a magnetising force OC is required to reduce the residual magnetism. The value of magnetising force is called coercive force of coercively. So the coactivity is a measure of the magnetising field required to destroy the residual magnetism in the specimen.

Further increasing of H in reverse direction gives a saturation point D. The point D is symmetrical to A. If now the field is increased in steps a curve DEFA is obtained.

It can be observed from the figure that throughout the cycle ABCDEFA, the intensity of magnetisation M lags behind the magnetising field H. Thus the

lagging of intensity of magnetisation behind the magnetic field is called hysteresis.

The closed curve ABCDEFA which represents a cycle of magnetisation of the specimen is known as the "hysteresis curve (or loop)" of the specimen.

5.7 Work done in taking unit volume of a magnetic material through a complete cycle of magnetisation

According to Ewing's theory of molecular magnetism, a magnetic material even in the unmagnified condition consists of an indefinitely large number of molecular magnets endowed with define polarity. When a magnetizing field is applied, the molecular magnets align themselves in the direction of the field.

During this process, work is done by the magnetizing field in turning the molecular magnets against the mutual attractive forces. This energy required to magnetize a specimen is not completely recovered when the magnetizing field is turned off, since the magnetization does not become zero. The specimen retains some magnetization because some of the molecular magnets remain aligned in the new formation due to the group forces. To tear them out completely, a coercive force in the reverse direction has to be applied. Thus, there is a loss of energy in taking a ferromagnetic material through a cycle of magnetization. This loss of energy is called hysteresis loss and appears in the form of heat.

Consider a magnetic material having n molecular magnets per unit volume. Let m be the magnetic moment of each magnet and θ the angle which its axis makes with the direction of magnetizing field H.

The magnetic moment m of the molecular magnet can be resolved into a component m cos θ in the direction of H and m sin θ perpendicular to H. The component m cos θ alone contributes to the magnetising field and the component m sin θ has no effect on the magnetisation of the specimen.

If M be the intensity of magnetisation, then

$$M=\Sigma m \cos \theta \qquad \ldots\ldots(1)$$

Differentiating Eq. (1), $dM = d(\Sigma m \cos \theta) = -\Sigma m \sin \theta \, d\theta$ ....(2)

When M increases to M + dM, θ decrease to θ - d θ

The work done by the field in decreasing θ by $d\theta$ is given by

$$dW = C d\theta \qquad \ldots\ldots(3)$$

here, C = torque for unit deflection = $\mu_0 \, m \, H \sin \theta$

$$dW = \mu_0 \, mH \, sin\theta \; X \; (-d\theta) = -\mu_0 mH \; sin\theta \; d\theta) \qquad \ldots..(4)$$

The work done by the applied field is

$$=\Sigma\, dW = \mu_0 H\, X\, (-\Sigma m \sin\theta\, d\theta)$$
$$=\mu_0 H\, X\, dM \quad (from\ Eq.2)$$

Thus work done by the magnetizing field per unit volume of the material for completing a cycle is

$$W = \oint \mu_0 H\, dM = \oint H\, \mu_0\, dM \qquad .....(5)$$

Now, B $= \mu_0(H + M)$. For ferromagnetic, M>>H. So B $= \mu_0\, M$.

i.e., dB $= \mu_0 dM$

from Eqs.(5) and (6).

$$W = \oint H\, dB \qquad ....(7)$$

The area of the B-H loop or μ_0 times the area of the M-H loop gives the energy spent per cycle. When H is in Am^{-1} and b is in Wbm^{-2}, the energy is in joules per cycle per m^3 of the material.

5.8 Area of the hysteresis loop

Let ABCDEFA represent the M-H curve of the materials (Fig 5.16 & 5.17)

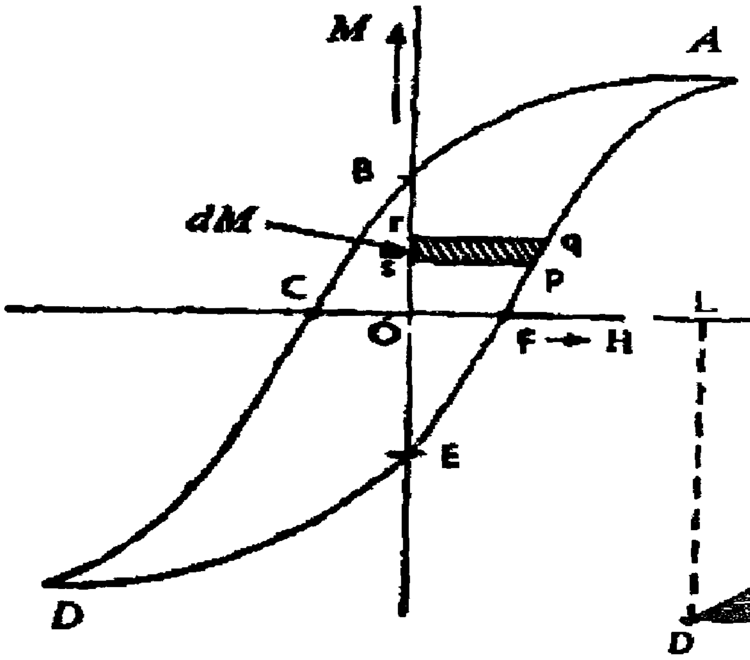


Fig 5.16

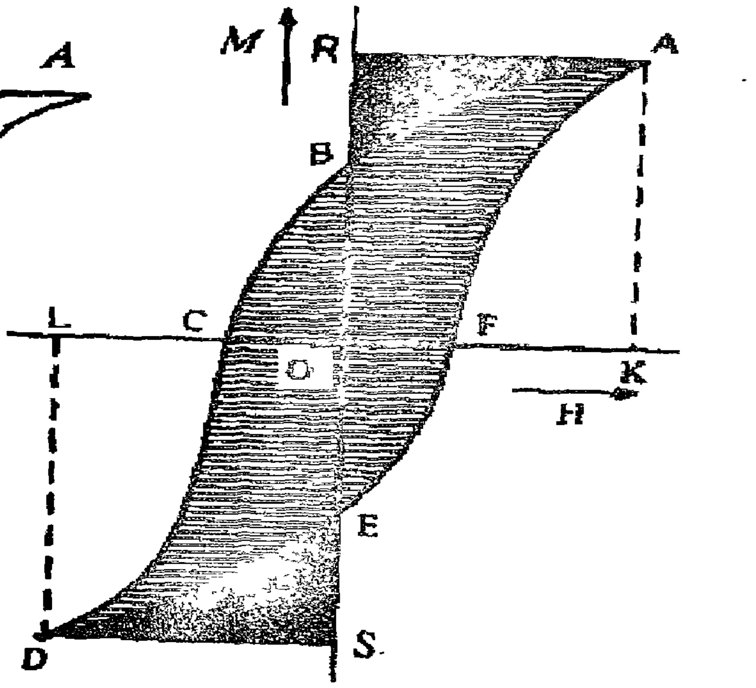


Fig 5.17

Consider two very neat points p and q on the curve.

At p, the value of the magnetising field H is represented by sp.

The increase in the value of M is dM. The amount of work done on the specimen to increase the intensity of magnetisation by dM.

$$= \mu_0 \oint H\, dM$$

This is equal to μ_0 times the area of the small rectangular strip pqrs. Similarly, the whole cycle can be imaging to be made up of small rectangular strips and the total amount of work done on the specimen when H increases or by the specimen when H decreases to zero can be calculated.

(i) The work done on the specimen when the value of H increases from zero to K (Fig 5.23).

$$= \mu_0 \int_E^R HdM = \mu_0(area\ EFARBOE)$$

(ii) Work done by the specimen when H decreases to zero

$$= \mu_0 \int_R^B HdM = \mu_0(area\ ARBA)$$

(iii) Work done on the specimen when H increases from zero to L in the opposite direction.

$$= \mu_0 \int_B^S HdM = \mu_0(area\ BCDSEOB)$$

(iv) Work done on the specimen when H decreases to zero

$$= \mu_0 \int_S^E HdM = \mu_0(area\ DSED)$$

Total work done on the specimen per unit volume per cycle

$$= \mu_0[Area\ EFARBOE - Area\ ARBA + Area\ BCDSEOB - Area\ DSED]$$

$$= \mu_0[Area\ ABCDEFA]$$

$$= \mu_0\ times\ the\ area\ of\ the\ M - H\ loop$$

Hence, the work done on the specimen per unit volume when taken through a complete cycle of magnetisation is equal to μ_0 times the area of the hysteresis loop ABCDEFA.

5.9 Experiment to draw B-H curve (Ballistic method)

Circuit Description: A specimen of the given ferromagnetic material is taken in the form of a ring (Rowland ring).

The experimental arrangement is shown in Fig.5.18

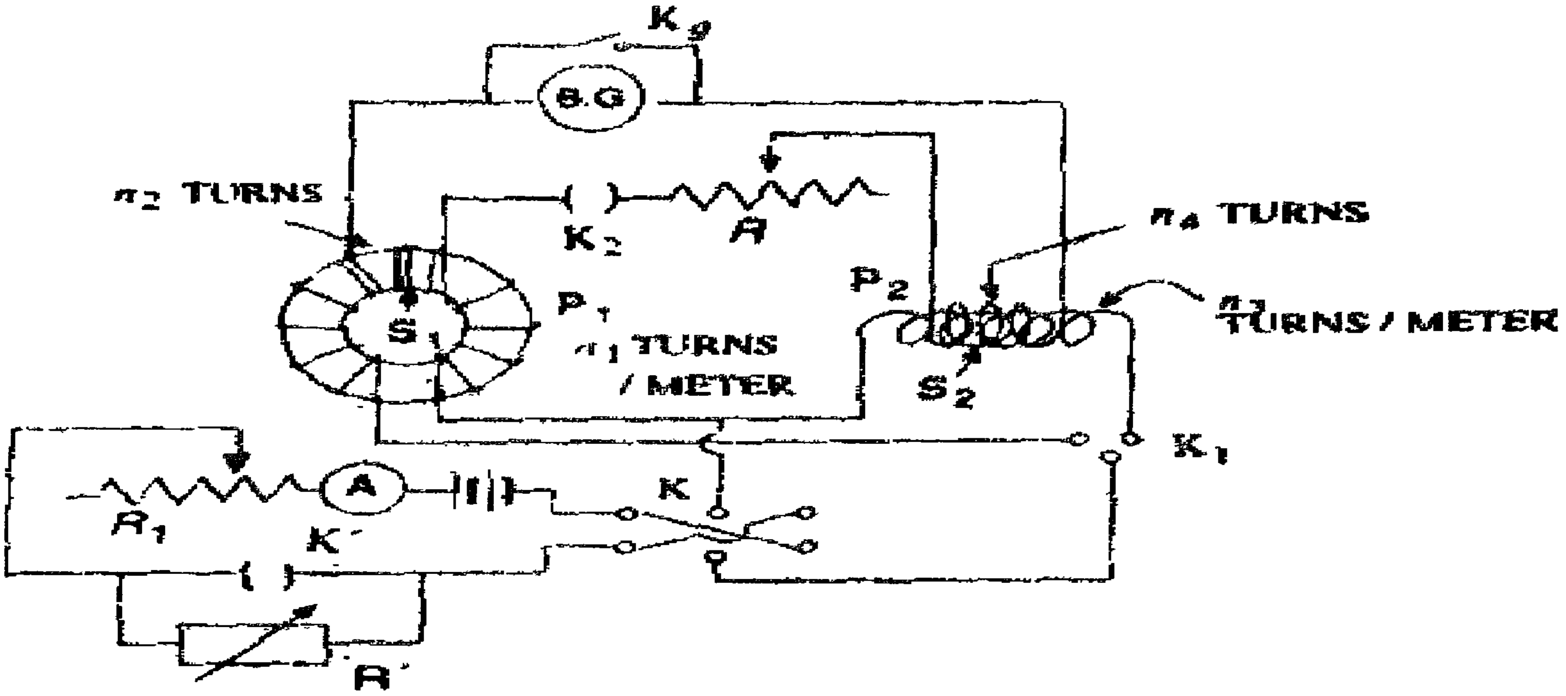


Fig 5.18

A primary coil P_1 is wound closely over the specimen ring. This winding is connected in series with a battery B, an ammeter A, a rheostat R_1 and a resistance R through a reversing key K and a two-way key K_1 A tap key K' connected across R' facilities either its inclusion or removal from the circuit. The secondary winding S_1 over the specimen consists of a few turns of closely wound wire. This winding S_1 is connected in series with a rheostat R, a ballistic galvanometer and the secondary winding S_2 of a standard solenoid through a key K_2, K_g is the damping key across the ballistic galvanometer. P_2 is the primary winding of the standard solenoid. The two ways key K_1 connects either P_1 or P_2 to the battery circuit.

Theory

Number of turns of the winding P_1	$= n_1$ turns per metre
Total number of turns of the winding S_1	$= n_2$ turns
Number of turns of the winding P_2	$=n_3$ turns per metre
Total number of turns of the winding S_2	$=n_4$ turns
Area of cross –section of the specimen	=A sq.metres
Area of cross-section of the standard solenoid	= a sq.metres

When the key k_1 is closed to the left, a current i passes through the magnetising coil P_1. The ring is magnetised.

The intensity of the magnetising field $= H=n_1i$ ....(1)

The magnetisation of the specimen develops a magnetic flux density P_2 inside the ring. Then,

The total flux linked with the secondary $= \varphi = n_2BA$

This is the change of flux in the secondary. It sets up an induced emf in the secondary circuit. If R is the total resistance of the secondary circuit, then the charge passing through the ballistic galvanometer.

$q=n_2BA/R.$

U θ is the first throw of the ballistic galvanometer coil, then

$$q = n_2BA/R = K\,\theta\left(1+\frac{\lambda}{2}\right). \qquad ......(2)$$

Here , K is the ballistic constant and

λ the logarithmic decrement of the ballistic galvanometer.

To eliminate K and λ.

A known current I is passed through the primary of the standard solenoid by closing the key K_1 to the right.

Magnetic flux linked with the secondary = $\varphi = \mu_0 n_3 i\, a n_4$ Wb

This change in the magnetic flux sends a charge $q' = \frac{\mu_0 n_3 n_4 ia}{R}$

Through the galvanometer

If θ' is the first throw in the galvanometer coil, then

$$q = \frac{\mu_0 n_3 n_4 ia}{R} = K\theta'\left(1+\frac{\lambda}{2}\right) \qquad ......(3)$$

Dividing Eq.(2) by Eq.(3) we have

$$\frac{n_2BA}{\mu_0 n_3 n_4 ia} = \frac{\theta}{\theta'}$$

or

$$B = \frac{\mu_0 n_3 n_4 ia}{An_2}\frac{\theta}{\theta'}\ Wb/m^2 \qquad ...(4)$$

Eq.(4) gives the magnetic induction B induced in the specimen corresponding to the magnetic intensity H, given by Eq.(1).

Procedure: The key K_1 is first closed to the left and the resistances R_1 and R' are decreased until on closing the commutator K, the galvanometer gives a full-scale deflection from the zero. The current required to do this is noted and is used as maximum current in the main experiment.

The residual magnetism in the specimen is reduced to zero as follows. The galvanometer circuit is first broken and the resistance R_1 and R' are reduced to the minimum. The current passing through the primary of the ring solenoid is then reversed many times by means of the commutator K and R and R' are gradually increased until the current which is reversed is very small.

The galvanometer is again put in the circuit by closing key K_2. The key K' is closed and resistance R_1 is given a value corresponding to the maximum current. The commutator K is closed to the right and the first throw θ_1 of the galvanometer is noted. The current i_1 is also noted from the ammeter. The values of B_1 and H_1 are calculated by using Eqs. (4) and (1) respectively. The corresponding point of the B-H curve is (Fig 5.19)

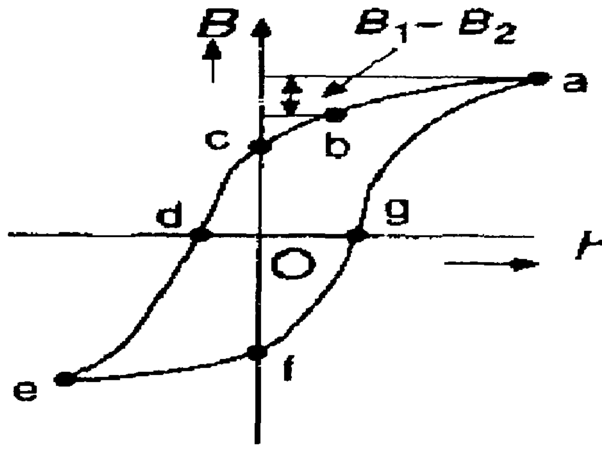


Fig 5.19

The galvanometer circuit is again broken and the specimen is again demagnetized by reversing rapidly the commutator K as described before. The ballistic galvanometer is again put in the circuit. Now R is given a small value and K is opened. The magnetising force is thereby decreased to H_2, producing a ballistic throw θ_2 in the galvanometer. This throw corresponds to a decrease in induction $B_1 - B_2$.

The value of magnetising field H_2 is calculated by noting ammeter reading. The corresponding point on the graph is denoted by the point b. this process is repeated by gradually increasing R, until current and hence H becomes zero. The graph corresponding to these readings is ac. After each measurement, the specimen is returned to the state a by the reversal of maximum current. Hence point a works as the reference point.

The key K is now closed and commutator is reversed several times and finally left to the right. R is given a large value and galvanometer is again put in the circuit. The commutator K is then thrown over to the left and at the same time K is opened, so that the current is reversed and at the same time made of small value. This gives a point on the part cd of the curve. The starting point is again a and change in magnetic inductions is measured every time. The process is repeated in many steps until finally R is zero when the point e on the curve is reached.

The part *efga* can be drawn by symmetry, or by repeating the experiment using e as the reference point and leaving the commutator now on the left.

The two-way key K_1 is closed to the right. A known current i is passed through P_2. The corresponding throw θ in the B.G., is noted. This auxiliary experiment is used to calculated B from Eq.(4)

Ferromagnets & Ferrimagnets:

Transition metal Fe, Co, and Ni; rare earth metals such as Gd and a few oxides such as CrO_2 and ErO display ferromagnetism.

a) Ferrimagnetic Materials and their Properties:

In ferrimagnetic materials (also called ferrites) such as Mn $Fe_2 O_4$, the magnetic moments of adjacent ions are anti parallel and of unequal strength (Fig 5.20). So there is a finite net magnetisation. By suitable choice o rare-earth ions in the ferrite lattices it is possible to design ferrimagnetic substances with specific magnetization for use in electronic components.

The general formula of ferrites may be written in the simple form as Me^{+2} Fe_2^{+3} O^4 , where Me^{+2} represents divalent metallic ions such as Fe^{+2}, Co^{+2}, Mn^{+2}, Zn^{+2}, Cd^{+2}, Mg^{+2}, etc.

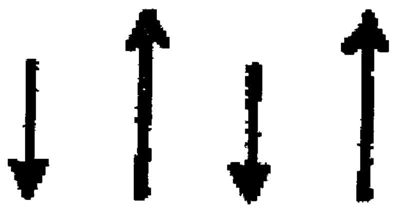


Fig 5.20

Properties:

1. These are metal oxides, but not metals
2. These material exhibit hysteresis property
3. Fig 5.21 shows the variation of susceptibility (χ) with temperature for ferrimagnetic materials.

   Magnitude of susceptibility is very large and positive.

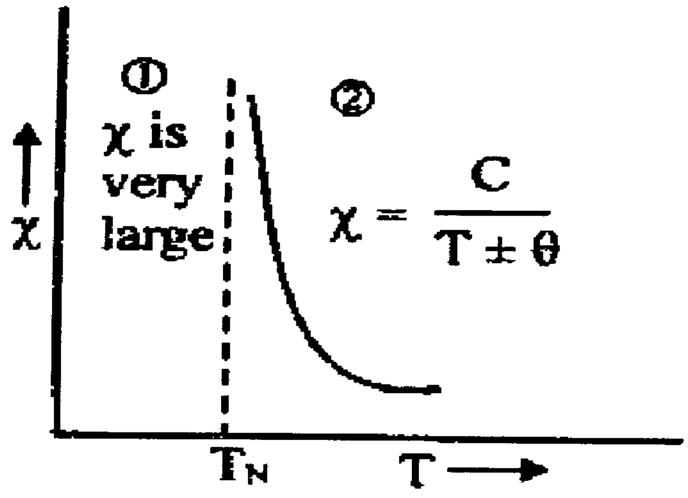


Fig 5.21

Temperature dependence of susceptibility.

At T >T_N (Neel temperature),$\chi = \frac{C}{T \pm \theta}$

4. They are insulators with very high resistivity ($\sim 10^{12}\ \Omega\ cm$). As a consequence, there will be no eddy current loss as usually noted with iron, at high frequencies.

5. High microwave dielectric constant (~ 10 to 12) (low dielectric loss).
6. A high magnetic permeability (500-1000) for the mixed Ni-Zn ferrites ($Ni_{0.36}$ $Zn_{0.64}$ Fe_2 O_4) at low frequencies and ~10 at high frequencies (≥ 300 MHz).
7. Ferrites show the same type of hysteresis curve, domain structure and motion similar to ferromagnetic; but the coercive force and saturation magnetisation (though appreciable) is smaller than that of ferromagnetic materials.
8. They have a Curie temperature that varies from 100°C to several hundred °C.
9. They are mechanically hard, brittle.
10. They are to be prepared using certain method, and it is not possible to cast them from the molten state because oxygen is then driven off from the material.
11. The resistivity depends on the preparation technique (and can be anywhere between 1 and 10^{12} Ω cm).

b) Applications of Ferrites (ferromagnetic materials)

1. Ferrites are used to produce low frequency ultrasonic waves by magnetostriction principle. Further these are used in the electromechanical transducers.
2. Ferrite rods are used in radio receivers (particularly in medium wave coil) to increase the sensitivity and selectivity of receiver.
3. Ferrites like Nickel Zine ferrites are used as cores in audio and T.V. transformers.
4. Since for ferrites eddy current loss and hysteresis loss are small at microwave frequencies, these are widely used in non-reciprocal microwave devices like gyrator, circulator and isolator.
5. Ferrites are also used in digital computers and data processing circuits. Normally here ferrites with rectangular hysteresis loops are used as magnetic storage elements.
6. Based on nonlinear tensor permeability property, ferrites can be used in devices for power limiting and harmonic generation.

5.10 Gouy's method for measurement of Susceptibility

Apparatus: It consists of an electromagnetic having parallel and flat pole pieces with a gap between them. The electromagnet provides a strong horizontal magnetic field of nearly 1Wb m^{-2}.

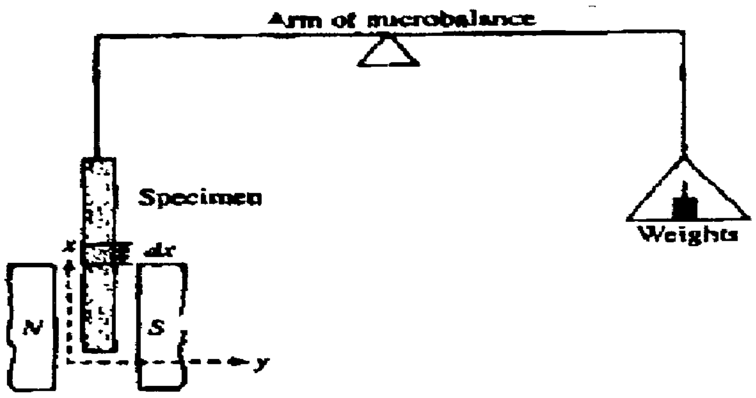


Fig 5.22

The sample under test is taken in the form of thin rod. It is suspended vertically from one arm of a sensitive micro balance in the magnetic field between the wedge shaped pole pieces of the electromagnet (Fig 5.22). the specimen is suspended in such a way that its lower end is near the mid-point of the magnetic field where the magnetic intensity H is large while its upper end is outside the field in the region of low intensity H_0. The cylindrical rod is then weighed by suspending it from a sensitive microbalance in the two cases:

(i) When field is off. (ii) when field is on

Theory: Let μ_1 *and* μ_2 be the permeability of air and specimen respectively. Consider a magnetic field intensity H in a region. Let V be the volume of the specimen. Hence the energy of magnetic flux in this region is $\frac{1}{2}\mu_1 H^2 V$ before inserting the specimen and $\frac{1}{2}\mu_1 H^2 V$, after inserting the specimen. The difference of these energies is equal to the work done in inserting the specimen.

$$\text{Work done} = \frac{1}{2}\,(\mu_2 - \mu_1)H^2 V \qquad .....(1)$$

This is stored as potential energy of field.

$$U = \frac{1}{2}\,(\mu_2 - \mu_1)H^2 V$$

We have, $\mu_r = 1 + \chi_m$ $or \frac{\mu}{\mu_o} = 1 + \chi_m$

$$\mu = \mu_o + \mu_o \chi_m$$

Let χ_1 *and* χ_2 *be the susceptibilities of* air and specimen. Then,

$$\mu_1 = \mu_o + \mu_o \chi_1 \; and \; \mu_2 = \mu_o + \mu_o \chi_2$$

$$\mu_2 - \mu_1 = \mu_o(\chi_2 - \chi_1) \qquad ...(2)$$

Substituting this in Eq.(1) we get

$$U = \frac{1}{2}\,\mu_o(\chi_2 - \chi_1)H^2V \qquad \text{... (3)}$$

Let F_x be the force acting on the specimen along X-axis. Then

$$F_x = -\frac{dU}{dX} = -\frac{d}{dx}\left\{\frac{1}{2}\mu_o(\chi_2 - \chi_1)H^2V\right\}$$

$$= -\frac{1}{2}\mu_o(\chi_2 - \chi_1)\frac{d}{dx}(H^2)V$$

If H_x, H_y and H_z are components of **H**, hen

$$H^2 = {H_x}^2 + {H_y}^2 + {H_z}^2$$

$$F_x = -\frac{1}{2}\mu_o(\chi_2 - \chi_1)\left\{\frac{d}{dx}\left({H_x}^2 + {H_y}^2 + {H_z}^2\right)\right\}V$$

$$= -\frac{1}{2}\mu_o(\chi_2 - \chi_1)V\left[2H_x\frac{dH_x}{dx} + 2H_y\frac{dH_y}{dx} + 2H_z\frac{dH_z}{dx}\right]$$

$$F_x = -\mu_o(\chi_2 - \chi_1)\left[H_x\frac{dH_x}{dx} + H_y\frac{dH_y}{dx} + H_z\frac{dH_z}{dx}\right]V \qquad \text{..(4)}$$

Now, considering X-axis to be vertical, the vertical force on a small element *dx* of rod of volume *dV*, at a distance *x* from origin

$$dF_x = -\mu_o(\chi_2 - \chi_1)\left[H_x\frac{dH_x}{dx} + H_y\frac{dH_y}{dx} + H_z\frac{dH_z}{dx}\right]dV$$

Let a be cross-sectional area of rod. Then,

dV = a dx

$$dF_x = -\mu_o(\chi_2 - \chi_1)\left[H_x\frac{dH_x}{dx} + H_y\frac{dH_y}{dx} + H_z\frac{dH_z}{dx}\right]adx \qquad \text{... (5)}$$

In the narrow gap between pole pieces, the magnetic flux will consist of straight lines direct from one pole face to another, in y direction say. Thus only the component H_x will be of significant magnitude while H_x and H_z will be negligible. Hence Eq.(5) is expressible as

$$dF_x = -\mu_o(\chi_2 - \chi_1)H_y\frac{dH_y}{dx}a\,dx \qquad \text{... (6)}$$

The total vertical force on the rod due to whole field variation along the length under the limit $H_y = H$ to $H_y = H_o$ is

$$F_x = -\mu_o(\chi_2 - \chi_1)a \int_H^{H_o} H_y \frac{dH_y}{dx}\, dx$$

$$= -\mu_o(\chi_2 - \chi_1)a \int_H^{H_o} H_y dH_y$$

$$= -\mu_o(\chi_2 - \chi_1)a \left[\frac{H^2{}_y}{2}\right]_H^{H_o}$$

$$= -\frac{1}{2}\mu_o(\chi_2 - \chi_1)a\ (H_0{}^2 - H^2)$$

$$= -\frac{1}{2}\mu_o(\chi_2 - \chi_1)a\ (H^2 - H_0{}^2) \quad \text{..... (7)}$$

Let m1and m_2 be the weights to counterpoise the force on rod with magnetic field off and on respectively. Then,

$$F_x = (m_2 - m_1)g \quad \text{..... (8)}$$

Equating (7) and (8), we get

$$(m_2 - m_1)g = \frac{1}{2}\mu_o(\chi_2 - \chi_1)a\ (H^2 - H_0{}^2)$$

$$(\chi_2 - \chi_1) = \frac{2(m_2 - m_1)g}{\mu_o a(H^2 - H_0{}^2)} \quad \text{.... (9)}$$

H_o and H can be measured by the flux meter. Thus by putting the susceptibility of air χ_1 and other known quantities in the above equation, χ_2 is calculated

5.11 Maxwel's Equations In Material Media

The differential form of Maxwell's four equations is given below

$$\nabla.\mathbf{D} = \rho \quad \text{......(1)}$$

$$\nabla.\mathbf{B} = 0 \quad \text{.....(2)}$$

$$\nabla \times \mathbf{E} = -\frac{\partial B}{\partial t} \quad \text{......(3)}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial D}{\partial t} \quad \text{..... (4)}$$

D = electric displacement vector

ρ = volume density of charge

B = Magnetic field induction

E = electric field intensity

H = magnetic field intensity

J = current density

For a linear, isotropic and homogeneous medium the *constitutive relations* are given by the following equations:

$$\mathbf{D} = \varepsilon \mathrm{E} \qquad \ldots\ldots(5)$$

$$\mathbf{B} = \mu \mathbf{H} \qquad \ldots\ldots(6)$$

$$\mathbf{J} = \sigma \mathbf{E} \qquad \ldots\ldots(7)$$

Here, ε, μ and σ denote respectively the dielectric permittivity, magnetic permeability and conductivity of the medium respectively.

Derivation of Maxwell's Equations:

1. Let us consider a surface S bounded by a volume V in a dielectric medium. In a dielectric medium total charge consists of free charge plus polarisation charge.

Let ρ and ρ_p be the charge densities of free charge and polarisation charge at a point in small volume element dV. Then Gauss law can be expressed as

$$\int_S E.ds = \frac{1}{\varepsilon_0}\int_V (\rho + \rho_p)dV \; But \; \rho_p = -div\, \boldsymbol{P}$$

$$\int_S E.ds = \int_V \rho\, dV - \int_V div\, \boldsymbol{P}\, dV \qquad \ldots\ldots(1)$$

But from Gauss divergence theorem

$$\int_S \varepsilon_0 E.ds = \int_V div\,(\varepsilon_0 E)dv$$

Therefore Eq.(i) gives

$$\int_V div\,(\varepsilon_0 E)dV = \int_V \rho\, dV - \int_V div\, \boldsymbol{P}\, dV$$

$$\int_V div\,(\varepsilon_0 E + P)dV = \int_V \rho\, dV$$

$$i.e., \int_V div\, D\, dV = \int_V \rho\, dV \qquad (since\ D = \varepsilon_0 E + P)$$

$$\int_V (div\, D - \rho)dV = 0$$

As volume is arbitrary, we must have

$$\text{div D} - \rho = 0$$

$$\text{i.e., div D} = \rho \qquad \text{....(2)}$$

2. We know that isolated magnetic poles do not exist. This fact is also expressed by saying that magnetic lines of force are, in general, closed curves. Because of the magnetic flux leaving and entering any closed surface is always same. In other words, the total magnetic induction flux over any closed surface is always zero, i.e

$$\int_S \boldsymbol{B}.\boldsymbol{dS} = \mathbf{0}$$

Using Gauss's theorem we change surface integral into a volume integral

$$\int_v (div\boldsymbol{B})dV = \mathbf{0}$$

Since the above statement is true for any arbitrary volume, V, it is only possible if the integrand itself is zero, i.e.,

$$\text{div } \mathbf{B}=0$$

$$\text{or} \quad \nabla.\boldsymbol{B} = o$$

3. By Faraday's law of electromagnetic induction, the induced emf in a closed loop is

$$e = -\frac{\partial \varphi}{\partial t} \qquad \text{..... (1)}$$

By definition, the magnetic flux φ over any arbitrary surface S is,

$$\varphi = \int_S \boldsymbol{B}.\boldsymbol{dS}$$

$$\text{Therefore,} \quad e = -\frac{\partial}{\partial t}\left[\int_S \boldsymbol{B}.\boldsymbol{dS}\right] = -\int_S \frac{\partial B}{\partial t}.\boldsymbol{dS} \qquad \text{.... (2)}$$

(∴ Surface S is fixed and hence only **B** is a function of time)

Again e.m.f., by definition, is the work done in carrying a unit charge round a closed loop. If C is the loop constituting the surface S, then

$$e = \int_C \boldsymbol{E}.\boldsymbol{dl} \qquad \text{.... (3)}$$

Where **E** is the electric field intensity at a point where elementary loop **dl** is located

Comparing Equations (2) and (3), we get

$$\int_C \boldsymbol{E}.\boldsymbol{dl} = -\int_S \frac{\partial B}{\partial t}.\boldsymbol{dS} \qquad \text{....(4)}$$

$$from\ Stoke'stheorem, \int \boldsymbol{E}.\boldsymbol{dl} = \int_s curl\ \boldsymbol{E}.\boldsymbol{dS}$$

Eq(4) becomes, $\int_s curl\, \boldsymbol{E}.\boldsymbol{dS} = -\int_S \frac{\partial B}{\partial t}.\boldsymbol{dS}$

Since surface is arbitrary, curl $\boldsymbol{E} = -\frac{\partial B}{\partial t}$

$$\nabla \times E = -\frac{\partial \boldsymbol{B}}{\partial t} \qquad .....(5)$$

4. According to Ampere's law, the work done in carrying a unit magnetic pole once around a closed arbitrary path linked with the current is expressed by

$$\oint \boldsymbol{H}.\boldsymbol{dl} = \boldsymbol{I} = \int J.ds \qquad ....(1)$$

However, the use of Stock's theorem gives

$$\int_S (\nabla \times H).\boldsymbol{dS} = \int J.ds$$

or

$$\nabla \times H = J \qquad ....(2)$$

This equation is incomplete and accounts for steady currents only. But for varying electric fields, the current density should be modified. The difficulty with above equation is that if we take divergence of above equation, then,

$$\nabla.(\nabla \times H) = \nabla.J = 0 \qquad ......(3)$$

However, this fact is disallowed by Continuity Equation ,according to which

$$\nabla.J = -\frac{\partial \rho}{\partial t}$$

On this basis Maxwell added another current density $\boldsymbol{J'}$ ***to J*** to make Eq.(2) as.

$$\nabla \times \mathbf{H} = \mathbf{J} + \boldsymbol{J'} \qquad ....(4)$$

So that continuity equation too is satisfied

Now divergence of Eq.(4) givens

$$\nabla.(\nabla \times H) = \nabla.\mathbf{J} + \nabla.\boldsymbol{J'} = 0$$

or

$$\nabla.\boldsymbol{J'} = -\nabla.\mathbf{J} = \frac{\partial \rho}{\partial t} \qquad ....(5)$$

We know that $\nabla.\mathbf{D} = \rho$

Eq.(5) becomes, $\nabla.\boldsymbol{J'} = \frac{\partial}{\partial t}(\nabla.\mathbf{D})$

or

$$\nabla.\boldsymbol{J'} = \nabla.\frac{\partial \boldsymbol{D}}{\partial t}$$

or

$$\boldsymbol{J'} = \frac{\partial \boldsymbol{D}}{\partial t}$$

In this way we observe that the additional current density $\boldsymbol{J'}$ **is due to** time variations of electric displacement **D.** It is termed as displacement current density. According to Maxwell, it is just as effective as **J** in producing magnetic field.

Thus Eq.(4) is written as

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial D}{\partial t} \qquad \text{....(7)}$$

Physical Significance of Maxwell's Equations:

By means of Gauss's and Stoke's theorems we can put the field equations in integral form and hence obtain their physical significance.

1. Maxwell's first equation is $\nabla . \mathbf{D} = \boldsymbol{\rho}$
   Integrating this over an arbitrary volume V, we get

$$\int_V \nabla . \mathbf{D} \, dV = \int_V \boldsymbol{\rho} \, dV$$

But from Gauss theorem, we get

$$\int_S \mathbf{D} . dS = \int_V \boldsymbol{\rho} \, dV = q.$$

Here, q is the net change contained in volume V.S is the surface bounding volume V. Therefore, Maxwell's first equation signifies that:

The total electric displacement through the surface enclosing a volume is equal to the total electric charge within the volume.

2. Maxwell's second equation is $\nabla . \mathbf{B} = 0$
   Integrating this over an arbitrary volume V, we get

$$\int_V \nabla . \mathbf{B} = 0.$$

Using Gauss divergence theorem to change volume integral into surface integral, we get

$$\int_S \mathbf{B} . d\mathbf{S} = 0.$$

Maxwell's second equation signifies that:

*The total outward flux of magnetic induction **B** through any closed surface S is equal to zero.*

3. Maxwell's third equation is $\nabla \times \mathbf{E} = -\frac{\partial B}{\partial t}$

Integrating this equation over a surface S, bounded by a curve C, we get

$$\int_S (\nabla \times \mathbf{E}).d\mathbf{S} = -\int_S \frac{\partial \boldsymbol{B}}{\partial t}.d\mathbf{S}$$

Converting the surface integral of left hand side into line integral by Stoke's theorem, we get

$$\oint_C \boldsymbol{E}.d\boldsymbol{l} = -\int_S \frac{\partial \boldsymbol{B}}{\partial t}.d\mathbf{S}$$

Maxwell's third equation signified that:

The electromotive force[e.m.f.e $=\int_C$ **E.dl]** *around a closed path is equal to negative rate of change of magnetic* ***flux linked with the*** *path [since magnetic flux* $\varphi = \int_S \boldsymbol{B}.d\mathbf{S}$

4. Maxwell's fourth equation is

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \boldsymbol{D}}{\partial t}$$

Taking surface integral over surface S bounded by curve C, we obtain

$$\int_S \nabla \times \mathbf{H}.d\mathbf{S} = \int_S \left(\boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}\right).d\mathbf{S}$$

Using Stoke's theorem to convert surface integral on L.H.S of above equation into line integral, we get

$$\oint_C \boldsymbol{H}.d\boldsymbol{l} = -\int_S \left(\boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}\right).d\mathbf{S}$$

Maxwell's fourth equation signified that:

The magnetomotive force (m.m.f $=\oint_C \boldsymbol{H}.d\boldsymbol{l}$*) around a closed path is equal to the condition current plus displacement current through any surface bounded by the path.*

5.12 Types of Currents

Ampere's law $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$ does not hold good for time varying fields. To remove this difficulty, Maxwell modified the above equation by introducing the concept of displacement current.

Putting $\mathbf{B}=\mu_0\mathbf{H}$, we get

$$\nabla \times \mathbf{H} = \mathbf{J}$$

Taking divergence of this equation, $\nabla.(\nabla \times \mathbf{H}) = \nabla.\mathbf{J}$

But the divergence of the curl of a vector is always zero

$$\nabla.\mathbf{J} = 0$$

This means that the divergence of current density is zero.

This is not true for time – varying fields.

From Gauss's law, $\nabla.\mathbf{E} = \rho/\varepsilon_0$

Differentiating, $\nabla.\frac{\partial E}{\partial t} = \frac{\partial \rho}{\partial t}$

Adding $\nabla.\mathbf{J}$ to both sides, we have

$$\nabla.\mathbf{J} + \varepsilon_0 \nabla.\frac{\partial E}{\partial t} = \nabla.\mathbf{J} + \frac{\partial \rho}{\partial t}$$

According to general equation of continuity, $\nabla.\mathbf{J} + \frac{\partial \rho}{\partial t} = 0$

$$\nabla.\mathbf{J} + \varepsilon_0 \nabla.\frac{\partial E}{\partial t} = 0$$

or

$$\nabla.\left(\mathbf{J} + \frac{\partial D}{\partial t}\right) = 0$$

$$(\mathbf{D}=\varepsilon_0\mathbf{E})$$

$\left(\mathbf{J} + \frac{\partial D}{\partial t}\right)$ is the *total current density.* Maxwell pointed out that this should replace **J** in Ampere's law

Hence the modified form of Ampere's law is

$$\nabla \times \mathbf{H} = \left(\mathbf{J} + \frac{\partial D}{\partial t}\right) \quad .....(5)$$

The first term on the R.H.S of Eq.(5) represents the conduction current density **J.**

The term $\frac{\partial D}{\partial t}$ is called displacement current. It is the time rate of change of the electric displacement. The second term is better termed as displacement current density $\mathbf{J_d}$. It shall have the same effect as the true current but shall be effective only when E is a time-varying field. Therefore, Eq.(5) becomes.

$$\nabla \times \mathbf{H} = (\mathbf{J} + \mathbf{J_d})$$

5.13 Displacement current

Maxwell introduced the new concept of displacement current. Faraday discovered that a changing magnetic field produces an electric field. Maxwell pointed out that changing electric field produces a magnetic field.

Consider the plates of a parallel plate capacitor connected to the terminals of a battery. The conduction current from the battery gradually charges the capacitor plates. Until the voltage across the capacitor becomes equal to the battery voltage, conduction current flows in the leads of the capacitor. Bu the conduction current is not continuous across the gap between the plates as there is no transfer of charge across the plates. But according to Maxwell, the changing electric field between the plates serves the purpose of conduction current inside the gap. The displacement current in the gap is found to be equal to the conduction current in the lead wires. This proves that the flow of current in a circuit is continuous.

5.14 Magnitude of Displacement current

Consider a parallel plate capacitor with free space between the plates. Let q be the charge given to one of the plates at any instant and an area.

Electric field between the plates

$$E = \frac{q}{\varepsilon_0 A}$$

$$E = \frac{q}{\varepsilon_0 A} \times \varepsilon_0 = \frac{q}{A}$$

Displacement current = $i_d = AJ_d$

$$= \mathbf{A}\frac{\partial \boldsymbol{D}}{\partial t}$$

$$= \mathbf{A}\frac{\partial \boldsymbol{D}}{\partial t}\left(\frac{q}{A}\right) = \frac{\partial \boldsymbol{q}}{\partial t} = i$$

Here, $\frac{\partial q}{\partial t}$ is the current flowing in the circuit at that instant

therefore $i_d = i$

Hence the displacement current in the gap between the capacity plates is equal to the conduction current in the lead wires. Thus the displacement current provides a continuous path for the charges across the capacitor.

5.15 Significance of Displacement current

The concept of displacement current was a great theoretical discovery as it rendered the relation ∇ x H = J between the current and magnetic field consistent with the continuity equation. The concept also led to altogether a new induction effect according to which a time varying electric field should give rise to magnetic effect as a time-varying magnetic field gives rise to electric field. Further, it helped to retain the notion that the flow of current in a circuit is continuous.

For example, let us consider the plates of a parallel plate capacitor connected to the terminals of a battery. The conduction current from the battery gradually charges the capacitor plates. Until the voltage across the capacitor becomes equal to the battery voltage, conduction current flows in the leads of the capacitor. Bu the conduction current is not continuous across the gap between the according to Maxwell, the changing electric field between the plates serves the purpose of conduction current inside the gap. The displacement current within the gap produced by changing electric field is found to be exactly equal to the conduction current as shown above, thus proving that the flow of current in a circuit is continuous.

5.16 Maxwell's equations in material media

There are four fundamental equations of electromagnetism known as Maxwell's equations which may be written in differential form as

$$\nabla . \mathbf{D} = \rho$$

$$\nabla . \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

and

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial D}{\partial t}$$

Here, ρ represents the charge density and **J** the current density. **E,D,B** and **H** represents the electric field intensity, electric displacement, magnetic induction and magnetic field intensity respectively. For a linear, isotropic and homogeneous medium,

$$\mathrm{D} = \varepsilon E, B = \mu H \text{ and } J = \sigma E.$$

5.17 Plane Electromagnetic Waves in free Space – Velocity of Light

Let us apply Maxwell's equations to develop wave equations for transverse electric and magnetic fields in free space. Let us consider a region where charge density ρ and current density **J** are both zero, i.e., for the space

$$\mu = \mu_0, \varepsilon = \varepsilon_0, \rho = 0 \; and \; \boldsymbol{J} = 0$$

Maxwell's equation then reduce to

$$\nabla . \boldsymbol{E} = 0 .... \quad (1)$$

$$\nabla . \boldsymbol{B} = 0 \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$\nabla \times \boldsymbol{B} = \varepsilon_0 \mu_0 \frac{\partial \boldsymbol{E}}{\partial t} \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

From Eq.(3), $\nabla \times (\nabla \times \mathbf{B}) = \varepsilon_0 \mu_0 \nabla \times \frac{\partial E}{\partial t} = \varepsilon_0 \mu_0 \frac{\partial}{\partial t} (\nabla \times \mathbf{E})$........................(5)

We have the vector identify

$$\nabla \times (\nabla \times \mathbf{B}) = \nabla(\nabla . \mathbf{B}) - \nabla^2 \mathbf{B}$$

$$= -\nabla^2 \mathbf{B} \qquad (\because \; \nabla . \mathbf{B} = \mathbf{0} \text{ from Eq. 2})$$

Eq (5) becomes $\nabla^2 \mathbf{B} = - \varepsilon_0 \mu_0 \frac{\partial}{\partial t} (\nabla * \mathbf{E}) = -\varepsilon_0 \mu_0 \frac{\partial}{\partial t} (-\frac{\partial B}{\partial t})$

$$\nabla^2 \mathbf{B} = \varepsilon_0 \mu_0 \frac{\partial^2 B}{\partial t^2} \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Similarly, $\nabla^2 \mathbf{E} = \varepsilon_0 \mu_0 \frac{\partial^2 E}{\partial t^2}$ ....(7)

Equations (6) and (7) represent wave equations governing electromagnetic fields **B** and **H** in free space

Equations (6) and (7) both have the form of the general wave equation with a speed

$$C = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

Now, $\mu_0 = 4\pi \times 10^{-7} Hm^{-1} \; and \; \varepsilon_0 = 8.854 \times 10^{-12} Fm^{-1}$

$$C = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = \frac{1}{\sqrt{4\pi \times 10^{-7} \times 8.854 \times 10^{-12}}} = 2.998 \times 10^{8} ms^{-1}$$

Which is the speed of light in free space. It thus follows that the field vector **E** and **B** can be propagated as waves in free space and they travel with the speed of light

Thus the velocity of EM waves in vacuum is the same as that of light in vacuum. This face led Maxwell to suggest that light is a form of em wave.

5.18 Propagation of Electromagnetic wave through a homogeneous, isotropic dielectric medium (non-conducting isotropic medium):

Maxwell's equations are

$$\nabla.\mathbf{D} = \rho$$

$$\nabla.\mathbf{B} = 0$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \boldsymbol{D}}{\partial t}$$

In an isotropic dielectric (or non – conducting isotropic medium)

$$\mathrm{D} = \varepsilon \boldsymbol{E}, \boldsymbol{B} = \boldsymbol{\mu H} \; and \; \boldsymbol{J} = \boldsymbol{\sigma E} = 0 \; and \; \boldsymbol{\rho} = 0$$

Therefore, Maxwell's equations in this case take the form

$$\nabla.\mathbf{E} = 0 \quad \text{....(1)}$$

$$\nabla.\mathbf{H} = 0 \quad \text{.........(2)}$$

$$\nabla \times \mathbf{E} = -\boldsymbol{\mu}\frac{\partial \boldsymbol{H}}{\partial t} \quad \text{...(3)}$$

$$\nabla \times \mathbf{H} = \varepsilon\frac{\partial \boldsymbol{E}}{\partial t} \quad \text{.....(4)}$$

Equation of Propagation of magnetic vector, H

Taking curl of Eq.(4), we get

$$\nabla \times (\nabla \times \mathbf{B}) = \varepsilon \nabla \times \left(\frac{\partial \boldsymbol{E}}{\partial t}\right)$$

Or

$$\nabla(\nabla.\mathbf{H}) - \nabla^2 \mathbf{H} \varepsilon \frac{\partial}{\partial t}(\nabla \times \mathbf{E}) \quad \text{.....(5)}$$

Putting values from Equation(2) and (3), we get

$$\nabla^2 \mathbf{H} = \mu\varepsilon \frac{\partial^2 H}{\partial t^2} \qquad ....(6)$$

Equation of Propagations of electric vector, E

Taking curl of Eq.(3), we get

$$\nabla \times (\nabla \times \mathbf{E}) = \nabla \times \left(-\mu \frac{\partial \boldsymbol{H}}{\partial t}\right)$$

$$\nabla(\nabla.\mathbf{E}) - \nabla^2\mathbf{E} = -\mu \frac{\partial}{\partial t}(\nabla \times \mathbf{H})$$

Putting values from Eqs.(1) and (4), we get

$$\nabla^2 \mathbf{E} = \mu\varepsilon \frac{\partial^2 E}{\partial t^2} \qquad .....(7)$$

Eqs (6) and (7) can be compared with the general wave equation

$$\nabla^2 \mathbf{u} = \frac{1}{\mathrm{v}^2}\frac{\partial^2 E}{\partial t^2}$$ Where v is the speed of wave.

$$\mathrm{V} = 1/\sqrt{\mu\varepsilon}$$

This means that the vector **E** and **H** are propagated in isotropic dielectric as waves with speed v given by

$$\mathrm{V} = \frac{1}{\sqrt{\mu\varepsilon}} \qquad ......(8)$$

Now , $\frac{1}{\sqrt{\mu_0 \varepsilon_0}} = c, speed\ of\ electromagnetic\ waves\ in\ free\ space$

Refractive index is $\mathrm{n} = \frac{c}{v} = \frac{\sqrt{\mu\varepsilon}}{\sqrt{\mu_0\varepsilon_0}} = \sqrt{\mu_r\varepsilon_r}$

In a non-magnetic medium $\mu_r = 1$

$$\mathrm{n} = \sqrt{\varepsilon_r}\ or\ n^2 = \varepsilon_r \qquad .....(9)$$

Eq.(9) is termed as Maxwell's relation and is experimentally verified in case of air, hydrogen, benzene and carbon etc., having non-polar molecules. But for substances containing polar molecules eg., water, glass etc.. ε_r is dependent of frequency and decreases with increasing frequency. Thus refractive index exhibits variations with frequency – a phenomenon termed as dispersion.

5.19 Let us sum-up

- **Magnetic induction (B)** :The net number of magnetic lines of force passing per unit area normally within the material medium when it is placed in an external magnetic (or magnetising) field is called the magnetic induction (**B**) within the sample.
- **Magnetisation M** of the material is defined as the magnetic dipole moment induced per unit volume of the material. Unit of **M** is Am^{-1}
- **The magnetic susceptibility** of a material is defined as the intensity of magnetisation acquired by the material for unit field strength.

- **Magnetic permeability (μ)** of a medium is defined as the ratio of magnetic induction to the intensity of the magnetising field. i.e., $\mu = \frac{B}{H}$; (For vacuum $x_m = 0$ *and* $\mu = \mu_0$)
- **Curie temperature:** The critical temperature above which a ferromagnetic material becomes a paramagnetic is called the **Curie temperature**.
- **Maxwell's equations:** There are four fundamental equations of electromagnetism known as Maxwell's equations which may be written in differential form as

$$\nabla.\mathbf{D} = \boldsymbol{\rho}$$
$$\nabla.\mathbf{B} = \mathbf{0}$$
$$\nabla \times \mathbf{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

and $$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \boldsymbol{D}}{\partial t}$$

- *The velocity of em waves in vacuum is the same as that of light in vacuum. This suggest that light is a form of EM wave.*

5.20 Unit –end exercises

1. Define magnetic induction. State the relation between B, H and I.
2. Define magnetic induction and intensity of magnetisation.
3. Define susceptibility and permeability of a magnetic material.
4. Define the three magnetic vectors namely, magnetic induction B, magnetisation I and magnetic intensity H. Establish the relation B $=\mu_0$ (H+M)
5. State the properties of diamagnetic substances.
6. Distinguish between the properties of para and dia magnetic substances.
7. What is Ferromagnetism? Give 2 examples for ferromagnetic substances.
8. Give the properties of ferro, para and dia magnetic substances.
9. Give a comparative study of Dia, Para and Ferro Magnetic substance.
10. Using electron theory of magnetism explains dia, para and ferromagnetism.
11. Describe the experimental determination of hysteresis loss using Ballistic method.
12. Give an account of Maxwell's equations. Discuss the significance of displacement current.
13. Derive Maxwell's equations for electromagnetic waves in vacuum.
14. Using Maxwell's equations, deduce the equation for an electromagnetic wave travelling through a dielectric medium.

5.21 Problems for discussion

1. A rod of magnetic material, 0.5 m in length has a coil of 200 turns wound over it uniformly. If a current of 2 ampere is sent through it, calculate (a) the magnetising field H, (b) the intensity of magnetisation M, (c) the magnetic induction B and (d) the relative permeability μ_r of the material. Given $\chi_m = 6 \times 10^{-3}$
2. An iron rod 0.2 m long. 10mm in diameter and of relative permeability 1000 is placed inside a long solenoid wound with 300 turns/metre. If a current of 0.5 ampere is passed through the rod, find the magnetic moment of the rod.
3. The magnetic susceptibility of a medium is 948×10^{-11} henry / metre. Calculate absolute permeability and relative permeability.
4. A material core has 10 turns per cm of wire wound uniformly upon it which carries a current of 2.0 ampere. The flux density in the material is 1.0 weber/ metre^2. Calculate the magnetising force and magnetisation of the material. What would be the relative permeability of the core?

5.22Answers for Check your progress and problems for discussion

Check your progress:

1. Magnetic permeability of a medium is defined as the ratio of magnetic induction top the intensity of magnetising field.
2. Sol: $\mu_r = (1+ \chi_m) = 1+2.14 \times 10^{-11}$
3. Yes, poles always occur in pairs just as every coin has two sides - a head and tail.
4. Ferromagnetic substances are those which are strongly magnetised by relatively weak magnetic field and in the same sense as the applied magnetic field.

5.23 Answers for problem for discussion:

1. Solution:

(a) $H = Ni/l = 800Am^{-1}$

(b) $M = \chi_m H = 4.8 \text{ A m}^{-1}$

(c) $B = \mu_0 (H+M) = 1.08 \times 10^{-3} \text{ Wb m}^{-2}$

(d) $\mu_r = B / \mu_0 H = 1.006$

2. Solution:

$H = Ni = 150$ ampere turns metre

$M = (\mu_r - 1) H = 149850$ ampere turns metre

$V = \pi r^2 l = 1.57 \times 10^{-5}\ m^3$

$M = M \times V = 2.353$ ampere metre^3

3. **Solution**

$\mu = \mu_0 (1 + \chi_m) = 1.2566 \times 10^{-6}$ H/m

$\mu_r = (1 + \chi_m) = 1.00000000948$

4. **Solution**

$H = Ni/l = 200$ amp-turns/metre.

$M = B/\mu_0 - H = 7.94 \times 10^{-5}$ amp-turns/metre

$\mu_r = B/\mu_0 H = 397.$

5.24 Suggested Readings:

1. Electricity and Magnetism - S Mahajan And A A Rangwala ,Tata McGrew Hill
2. Electricity and Magnetism - Dr.K.K.Tewari S.Chand & Co,2002.
3. Electricity and Magnetism with Electronics -D.N.Vasudeva S.Chand & Co,2002.
4. Electricity and Magnetism - Narayanamoorthy, Nagarathinam. 2nd Revised Edition

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